A Luna-tic Stablecoin Crash

Harald Uhlig[1]

[1] University of Chicago

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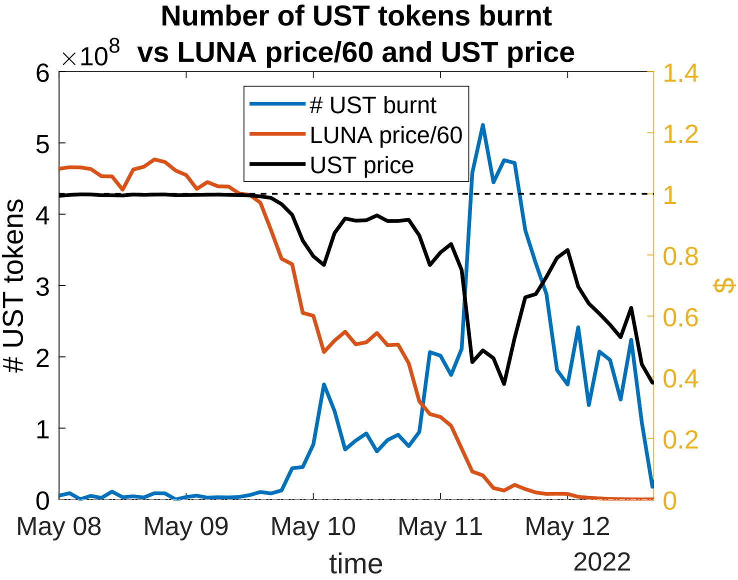
Background

- After remaining close to 1 US Dollar, the algorithmic stablecoin Terra UST lost more than 75 percent of its value in May, 2022, leading to a price collapse of the underlying LUNA token of 99.9 percent, an increase in LUNA supply by a factor of 19,000 and the erasure of more than 50 Billion U.S. Dollar.
- The system worked by allowing traders to convert a Terra UST coin into 1 U.S. Dollar worth of LUNA tokens and vice versa.
- Eventually, a sustained outflow or "burning" of UST coins into LUNA tokens resulted in the collapse of the system.
- The cryptocurrency market at large was affected. Barron's calculates that 600 Billion US Dollars were wiped out.
- Key figure cited in White House proposal on crypto regulation.
- Luna crash and recent FTX crash important driver of MiCA:
  - *"Crypto assets & DeFi have the potential to pose real risk to financial stability"*, Lagarde June 20
  - *"ECB president reiterates calls for 'MiCA II' in response to FTX collapse"*, Nov 28

The gradual unfolding of the crash



Objectives

Seek to **understand** these events **qualitatively** and **quantitatively**.

1. Theory: Luna.
   - Build a theory that generates a gradual unfolding of the LUNA crash, given a rate of UST Terra burning.
   - Provide closed-form solutions in a benchmark case.
   - Compare qualitatively to the data.
2. Theory: Terra UST.
   - Build a theory that justifies the rate of UST Terra burning.
   - Derive bounds for Terra UST underpricing.
3. Interpret the data.
   - Use the theory to back out the the theory-variables from the data to exactly justify the observations.

Method of Quantitative Interpretation

Compare to

- Identification
- Estimation
- Calibration

Pricing Cryptocurrencies

- NPV = 0? This is no different from fiat currencies.
- Benchmark: competing fiat currencies.
- "Exchange Rate Indeterminacy" (Kareken-Wallace, 1981):
  - **Kareken - Wallace, 1981**: In an OLG model with two perfectly substitutable monies, their exchange rate Q_t (or price of currency B in terms of currency A) is indeterminate and constant forever,

$$Q_t = \qquad\qquad\qquad Q_{t+1}$$

  - **Manuelli-Peck, 1990**: In a stochastic OLG model with two perfectly substitutable monies, the exchange rate Q_t satisfies

$$Q_t = E_t \left[\frac{v'(c_{2,t+1})}{u'(c_{1,t})} \frac{P_t}{P_{t+1}} Q_{t+1} \right]$$

    where $U_t = u(c_{1,t}) + E_t[v(c_{2,t+1})]$ and P_t is the price of date-t goods in terms of currency A.
  - **Schilling-Uhlig, 2019**: Q_t **is a risk-adjusted martingale**,

$$Q_t = \mathbb{E}_t \left[\mathcal{M}_{t+1} Q_{t+1} \right], \text{ where } \mathcal{M}_{t+1} = \frac{u'(c_{t+1}) \frac{P_t}{P_{t+1}}}{\mathbb{E}_t \left[u'(c_{t+1}) \frac{P_t}{P_{t+1}} \right]}$$

Challenges

- With KW, MP & SU:, shouldn't this have worked?
  - KW, MP & SU: Market Price Q_t of Luna is a martingale.
  - **Assume:** surprise movements in Q_t are uncorrelated with surprises regarding Terra UST inflows & outflows.
  - Result: the burning of Terra UST should have been absorbed by a higher market cap of Luna rather than its price collapse.
- One can fix that with **assuming correlations** of the needed kind. But why should they be there? Free parameter.
- Different perspective: **(1) assumption** for the evolution of **market cap**italization $m_t = Q_t M_t$.
- **Tension with KW, MP & SU**: given m_t and M_t, calculate Q_t.
- Imperfect substitutability of (crypto-)currencies? Terra UST as raison d'être for Luna?
- And how come, the **crash unfolded gradually**? Simple model & rational expectations: crash should be immediate.
- Solution is via **(2) assumption: continual hope for resurrection.**

1. Theory: Luna. An intro model

- Two periods: before burning, after burning.
- Initial Luna stock: M.
- Price: Q. Rational traders: same price in both periods.
- Burning of b UST.
- Final Luna stock:

$$\tilde{M} = M + \frac{b}{Q}$$

or

$$Q\tilde{M} = QM + b$$

- **(1) Assumption** for final **market cap** $\tilde{m} = Q\tilde{M}$.
- Solve for Q:

$$Q = \frac{\tilde{m} - b}{M}$$

- Need: $\tilde{m} > b$ for $Q > 0$. **Note: it may be:** $m < b$.
- Next: dynamics, gradual crash ...

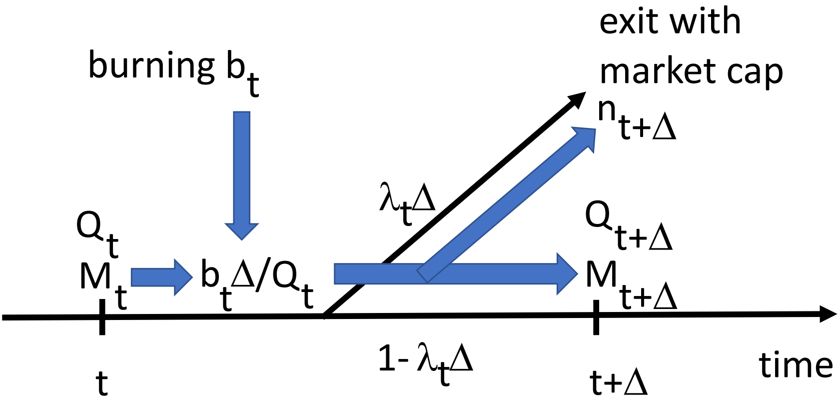
Gradual crash. TERRA UST peg: May 2022

Gradual crash. USDC peg: March 2023

1. Theory: Luna. Dynamics.

- At $t = 0$: “MIT shock”, burning b_t begins.
- Treat b_t as exogenous for this part.
- At t, and within the next time interval Δ,
  - burning of $b_t \Delta$ UST coins into $b_t \Delta / Q_t$ LUNA tokens.
  - **Assumption (2): continual hope for resurrection.** With exogenously given probability $\lambda_t \Delta$: the burning of UST stops and LUNA has an exogenously given market cap $n_{t+\Delta}$.
  - with probability $1 - \lambda_t \Delta$: the burning of UST continues.
- When LUNA price reaches $\epsilon > 0$: suspension of convertibility and exogenously given terminal market cap m_T for LUNA.
- Rational traders price LUNA tokens, taking dilution into account.
- For $\Delta \to 0$: system of ODEs.
- Closed-form solution, when $b_t \equiv b$, $\lambda_t \equiv \lambda$, $n_t \equiv n$.

Timeline


burning b_t
exit with
market cap
$n_{t+\Delta}$
$\lambda_t\Delta$
Q_t
M_t
$b_t\Delta/Q_t$
$Q_{t+\Delta}$
$M_{t+\Delta}$
$1-\lambda_t\Delta$
t
$t+\Delta$
time

Equations

Evolution of stock of tokens: forward in time.

$$M_{t+\Delta} = M_t + \frac{b_t \Delta}{Q_t} \tag{1}$$

Pricing equation: backwards in time.

$$Q_t = \frac{n_{t+\Delta}}{M_{t+\Delta}} \lambda_t \Delta + (1 - \lambda_t \Delta) Q_{t+\Delta} \tag{2}$$

Solve numerically?

Examine market cap $m_t = M_t Q_t$! Backward in time:

$$m_t + b_t \Delta = \lambda_t n_{t+\Delta} \Delta + (1 - \lambda_t \Delta) m_{t+\Delta} \tag{3}$$

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ODEs

$$\text{Define:} \quad y_t = \lambda_t \left(\frac{n_{t+\Delta}}{m_{t+\Delta}} - 1 \right) \tag{4}$$

$$m_{t+\Delta} - m_t = -\lambda_t (n_{t+\Delta} - m_{t+\Delta})\Delta + b_t\Delta = -y_t m_{t+\Delta}\Delta + b_t\Delta \tag{5}$$

$$Q_{t+\Delta} - Q_t = -y_t Q_{t+\Delta}\Delta \tag{6}$$

For $\Delta \to 0$:

$$\dot{m}_t = -\lambda_t (n_t - m_t) + b_t \tag{7}$$

$$\dot{Q}_t = -\lambda_t \left(\frac{n_t}{m_t} - 1 \right) Q_t \tag{8}$$

$$\dot{M}_t = \frac{b_t}{Q_t} \tag{9}$$

Exogenous: b_t, λ_t, n_t, m_T. T solves $Q_T = \epsilon$, given ϵ.

Define the relative burn rate

$$\alpha_t = \frac{b_t}{\lambda_t n_t} \tag{10}$$

A Simulation

Parameters:

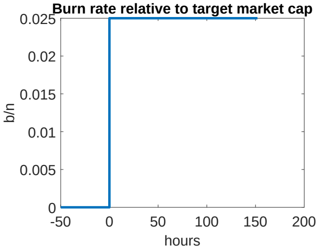
initial market cap:	m_t	$= 30$ for t < 0
initial price:	Q_t	$= 100$ for t < 0. Thus, $M_t = .3$ for t < 0
exit market cap:	n	$= 30$
exit rate:	λ	$= 0.05$
relative burn rate:	$\alpha = \frac{b}{\lambda n}$	$= \begin{cases} 0 & \text{for} t < t^* \\ 0.5 \text{ for } t \geq t^* \end{cases}$
stopping price:	$Q_T = \epsilon$	$= 0.1$
stopping market cap:	m_T	$= (1 - \kappa)n$ for the "stop decline rate"κ

Three scenarios:

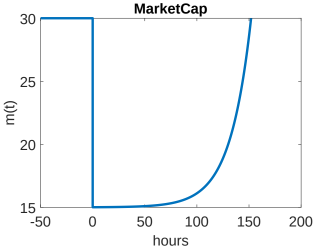
1. Burning start $t^* = 0$. Stop decline rate $\kappa = 0$, i.e. $m_T = n$.
2. Burning start $t^* = 50$. Stop decline rate $\kappa = 0$, i.e. $m_T = n$.
3. Burning start $t^* = 50$. Stop decline rate $\kappa = 0.9$, i.e. $m_T = n/10$.

$t^* = 0$. $\kappa = 0$ or $m_T = n$

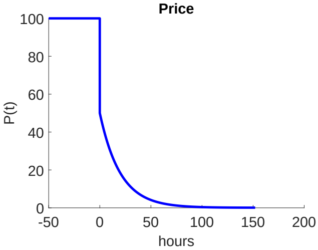
Burning rate of UST relative to n:



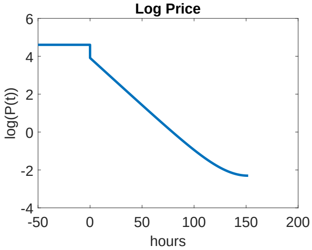
Market cap dynamics:



Price dynamics:

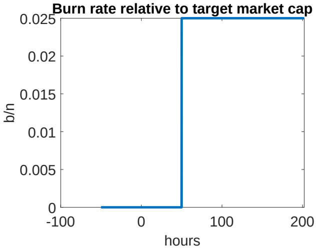


Log price dynamics:

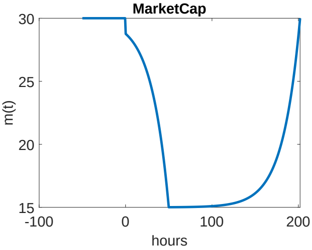


$t^* = 50$. $\kappa = 0$ or $m_T = n$

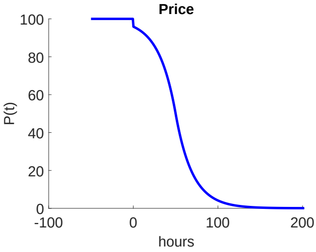
Burning rate of UST relative to n:



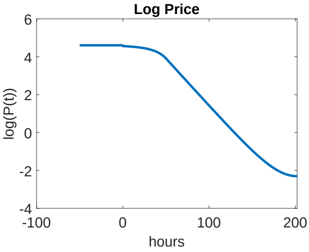
Market cap dynamics:



Price dynamics:



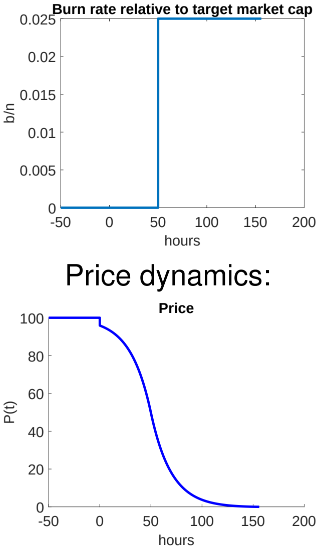
Log price dynamics:



$t^* = 50$. $\kappa = 0.9$ or $m_T = n/10$

Burning rate of UST relative to n: Market cap dynamics:

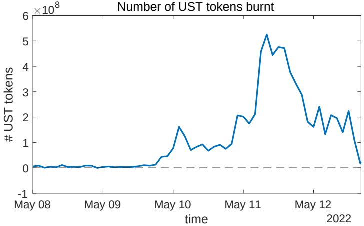
Price dynamics: Log price dynamics:



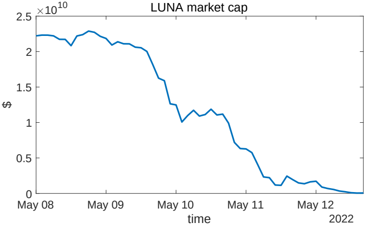
Looks like a stylized version of the data!

Data

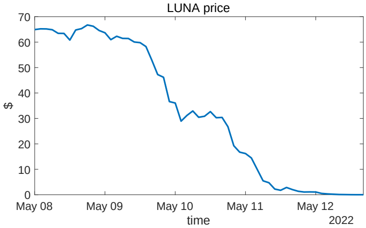
Burning of UST:



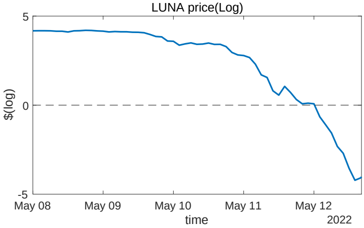
Market cap dynamics:



Price dynamics:



Log price dynamics:



3. Interpret the data

Method of quantitative interpretation:
use data to back out n_t, λ_t, m_T.

Calculations

$$\text{Accounting:} \quad M_{t+\Delta} = M_t + \frac{b_t \Delta}{Q_t^a} \tag{11}$$

$$\text{Define:} \quad x_t = -\frac{Q_{t+\Delta} - Q_t}{Q_{t+\Delta}\Delta} \tag{12}$$

Market capitalization is $m_t = Q_t M_t$ for all t. Thus,

$$m_{t+\Delta} - m_t = -x_t m_{t+\Delta}\Delta + \frac{Q_t}{Q_t^a} b_t \Delta \tag{13}$$

$$Q_{t+\Delta} - Q_t = -x_t Q_{t+\Delta}\Delta \tag{14}$$

Compare to theory:

$$\text{Define:} \quad y_t = \lambda_t \left(\frac{n_{t+\Delta}}{m_{t+\Delta}} - 1 \right) \tag{15}$$

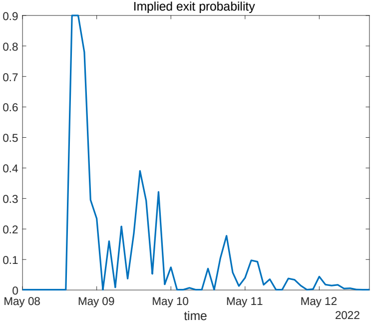
$$m_{t+\Delta} - m_t = -y_t m_{t+\Delta}\Delta + b_t \Delta \tag{16}$$

$$Q_{t+\Delta} - Q_t = -y_t Q_{t+\Delta}\Delta \tag{17}$$

Thus, one can infer y_t from the data, but not n_t and λ_t separately. Any data can be interpreted as arising from that solved-for y_t.

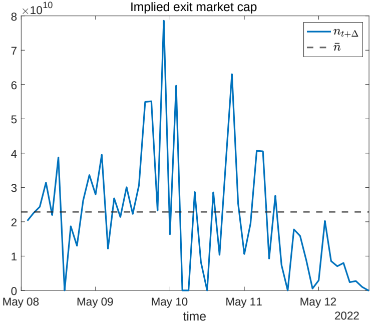
Data → Quantitative Interpretation

Scenario A: $n_t \equiv n$ and calculate exit probability λ_t



Restriction: $\lambda_t \geq 0$

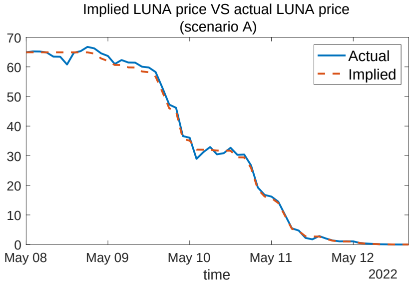
Scenario B: $\lambda_t \equiv \lambda$ and calculate exit market cap $n_{t+\Delta}$



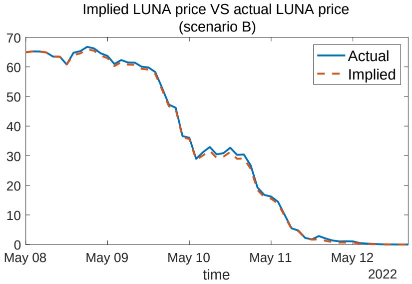
Restriction: $n_t \geq 0$.

Luna price: theory vs data

Scenario A:

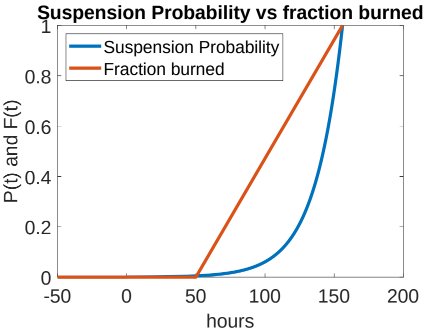


Scenario B:

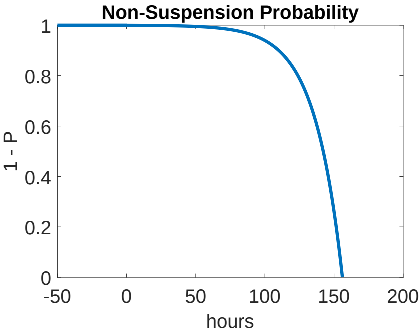


Burn rate and price floor: theory

Fraction UST burned over time:

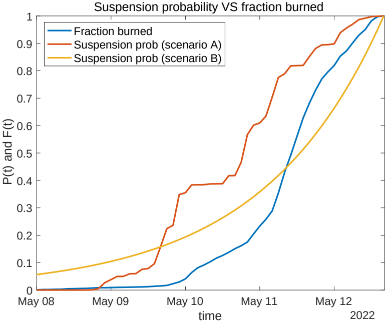


Price floor for UST coins:



Burn rate and price floor: data

Fraction UST burned vs P_t:



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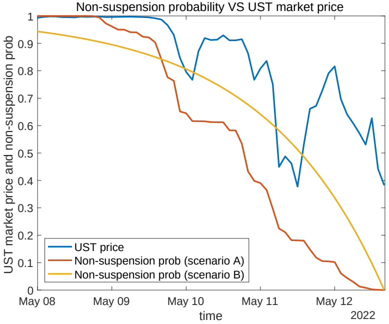
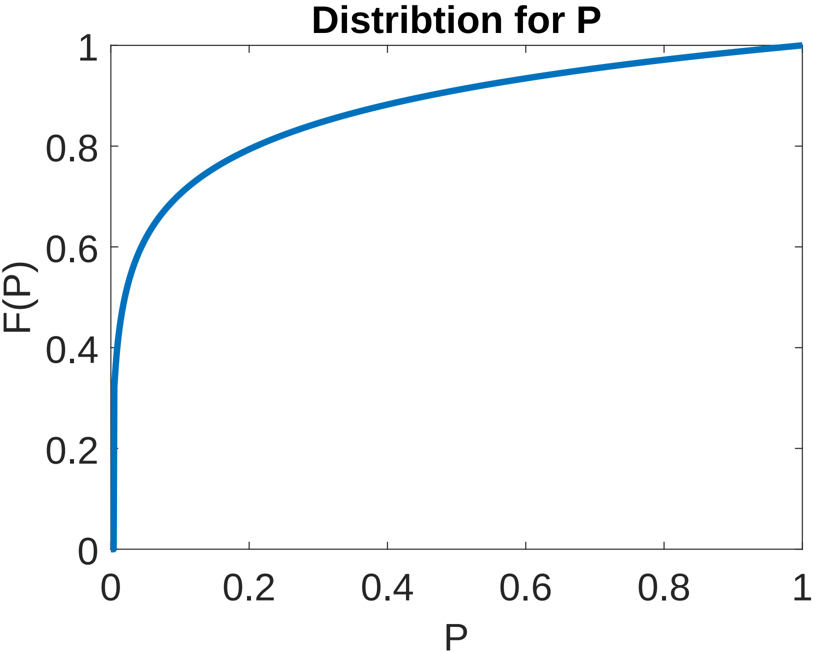


Distribution of threshold P for burning UST: theory.



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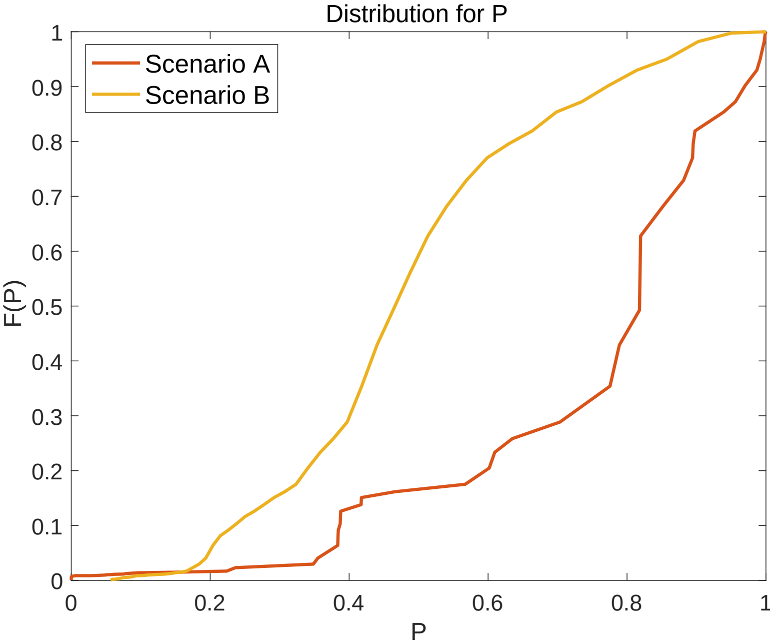

Conclusions

- Terra-Luna crash.
- Challenges:
  1. Kareken-Wallace etc: why a crash?
  2. Gradual rather than immediate?
- Theory:
  1. Impose assumptions on market capitalization.
  2. Impose that traders hope for resurrection, until price hits floor ϵ.
  3. ODEs. They can be solved.
  4. Closed-form solution in special case.
  5. Stylized version of the data.
- Method of quantitative interpretation:
  1. Use the data to back out theory variables.
  2. Examine the theory variables to understand the crash.
  3. Exit probability declined to zero as end drew near.
  4. Threshold probability of collapse, when burning UST was above 50% for more than 80% of UST holders.