The Coming Battle of Digital Currencies*

Lin William Cong† Simon Mayer§

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Abstract

We model the dynamic competition among national fiat currencies, cryptocurrencies, and Central Bank Digital Currencies (CBDCs), whereby a country's fiscal strength and currency strength are mutually reinforcing. The rise of cryptocurrencies hurts stronger fiat currencies (e.g., USD), but can benefit weaker fiat currencies by reducing competition from stronger ones. Countries strategically implement CBDCs in response to competition from emerging cryptocurrencies and other currencies. Our model reveals the following pecking order: Countries with strong but non-dominant currencies (e.g., China and India) have the highest incentives to launch CBDCs to gain technological first-mover advantage; countries with the strongest currencies (e.g., the United States) are the next in line and benefit from developing CBDC early on to nip cryptocurrency growth in the bud and to counteract competitors' CBDCs; nations with the weakest currencies forgo implementing CBDCs and adopt cryptocurrencies instead. Overall, fierce currency competition and the emergence of cryptocurrencies spur the development of CBDCs, generating valuable financial innovation. Our findings help rationalize recent developments in currency and payment digitization.

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†Cornell University SC Johnson College of Business and the FinTech Initiative. Email: will.cong@cornell.edu
§University of Chicago, Booth School of Business. Email: simon.mayer@chicagobooth.edu

In the past decades, privately owned payment systems (e.g., PayPal, M-Pesa, Alipay, and Square) have gained widespread popularity. The recent rise of cryptocurrencies, stablecoins, and decentralized has shown the potential to cause further disruption to the financial or monetary system as well as to payment technologies (Brunnermeier, James, and Landau, 2019; Adrian and Mancini-Griffoli, 2019; Cong, Li, and Wang, 2021a). Many countries and central banks around the globe react to these trends by actively researching or developing their own digital currencies, i.e., Central Bank Digital Currencies or in short CBDCs (Duffie, 2021; Duffie and Gleeson, 2021; Boar, Holden, and Wadsworth, 2020; Boar and Wehrli, 2021).[1] Also in the U.S., significant resources and efforts are being devoted to explore digital currency regulation and development, as exemplified in President Biden's Executive Order on digital currencies in March 2022.[2]

These developments raise many interesting and important questions. How do the emergence of cryptocurrencies and the development of CBDCs shape international currency competition? Which countries should develop CBDCs and when? How to react to other countries' digital currency initiatives? To examine these and related issues, we develop a dynamic model of currency competition among national fiat currencies, cryptocurrencies, and CBDCs, and provide a game-theoretic analysis of countries' strategies of digitizing money. Our theory helps rationalize international trends in payment and currency digitization, reveals a novel pecking order for CBDC development, and provides insights concerning the future of money and digital currency competition.

We study a model of currency competition between national fiat currencies of two countries, A and B, and one representative cryptocurrency C, which describes the broader cryptocurrency market including stablecoins pegged to fiat currencies. In each period, a representative OLG household is endowed with perishable consumption goods, which also serve as the numeraire. Importantly, the three currencies jointly fulfill the standard roles of money as (i) store of value allowing households to store endowments for desired consumption timing, (ii) medium of exchange generating a convenience yield, and (iii) unit of account. Households choose their holdings of A, B, and C to store

[1] For example, the Bank of Canada (Jasper Project) and the Monetary Authority of Singapore (Ubin Project) have tested the use of token-based CBDCs for cross-border wholesale settlements (Veneris, Park, Long, and Puri, 2021). The Hong Kong Monetary Authority and the Bank of Thailand have collaborated in a similar way (Inthanon-LionRock Project). European and Japanese central banks have also been actively researching digital currencies through Project Stella. China, in particular, has rolled out its digital currency program in 2020 and conducted US$13.8 billion of transactions in e-renminbi with 261 million users by the end of 2021, when the 2022 Beijing Winter Olympic Games was anticipated for test-driving the technology further (Rabouin, 2021). Some suspect China of waging a digital currency insurgency on the global financial system and the primacy of the dollar (Ehrlich, 2020), while others dismiss the impact (Eichengreen, 2021).

[2] See Fact Sheet March 09, 2022 from Statements and Releases, the White House.

their consumption goods over time, trading-off the currencies' convenience yield versus inflation and relative depreciation which compromise the store of value function.

National currencies A and B exhibit an endogenous debasement that decreases with the strength of their countries' economic fundamentals, captured in reduced form by the countries' expenses, including fiscal deficits, international trade costs, or debt service costs. For example, high expenses reflect weak economic fundamentals, cause a high inflation rate and/or depreciation relative to other currencies, and thus imply a weak national currency. We use A to denote the stronger country and its currency, which is more valuable in terms of the numeraire and can be viewed as the dominant or reserve currency (e.g., USD). Then, B represents a non-dominant competing currency (e.g., RMB or Euro). To incorporate that the U.S. dollar is often the currency of denomination for foreign debts (Maggiori, Neiman, and Schreger, 2020) and is the global unit of account for invoicing in international trade (Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller, 2020), we assume that countries' expenses increase with the strength of currency A.[3] Households' choice between national currencies induces a feedback and can lead to a vicious circle of inflation and depreciation for weaker currencies, which is exacerbated the stronger currency A is. As B depreciates, households substitute towards A, aggravating inflation and depreciation of currency B. Because the strength of currency A exacerbates inflation in country B, country A essentially imposes a pecuniary externality on the relatively weaker country B through a form of dollarization.

The representative cryptocurrency constitutes a viable substitute for national currencies as a store of value or medium of exchange. Households substitute toward cryptocurrency, when national currencies suffer from high inflation or the technology underlying cryptocurrency is efficient at facilitating transactions. That is, the absence of strong national fiat currencies implies a vacuum in the currency space which private cryptocurrencies fill. Importantly, the growth rate of cryptocurrency usage and convenience yield endogenously increase with adoption, giving rise to dynamic feedback and network effects: Higher cryptocurrency adoption today implies higher cryptocurrency adoption tomorrow which feeds back into adoption and prices today, causing the exponential growth of cryptocurrencies as witnessed in the recent past.

The cryptocurrency market essentially acts as a buffer zone amidst the battle between the two national fiat currencies and dampens the degree of dollarization and the vicious circle of debasement the weaker currency is exposed to. As the crypto sector grows and the household substitutes

[3]For theoretical foundations on the use of debt denominated in the dominant currency, see, among others, Eren and Malamud (2021); Eren, Malamud, and Zhou (2022).

toward crytpocurrency, the stronger currency A faces more competition from cryptocurrency and depreciates. Because the growth of the cryptocurrency market depends on the strength of currencies A and B, increasing the strength of B could benefit A by slowing the growth of the crypto-sector which in turn poses less competition to A. Meanwhile, the weaker currency B can benefit from the rise of cryptocurrencies, depending on whether the reduction of competition from A outweighs the increase in competition from cryptocurrencies. The model therefore rationalizes why countries with dominant currencies are more eager to regulate cryptocurrencies, whereas countries with the weakest currencies, such as El Salvador, do the opposite by officially adopting cryptocurrency.

We next consider the development of sovereign digital currencies, i.e., CBDCs. We model CBDC implementation in a technology-neutral manner without relying on any specific design, and stipulate that the launch of CBDC by country x increases the convenience yield of its currency x, which captures that CBDCs are a technological improvement upon traditional fiat currency (e.g., Duffie, Mathieson, and Pilav, 2021). As launching CBDC entails tremendous technological, legal, economic, and operational obstacles, we stipulate that a country successfully launches CBDC at a random time whose arrival rate increases with the country's endogenous (and costly) efforts (i.e., investments). In our model, countries exert such effort and develop CBDC so as to increase adoption, usage, or value of their currency upon the launch of CBDC. Countries' strategic decisions to implement CBDCs reflect competition from both cryptocurrencies and other national currencies and, in particular, depend on whether other countries have successfully launched CBDC yet.

The stronger country's incentives to launch CBDC mainly derive from the desire to compete with cryptocurrency. These incentives are relatively high when the cryptocurrency market is in its infancy, because then the launch of CBDC has the largest effect in reducing competition from cryptocurrencies. This effect gives rise to a "cryptocurrency kill zone" that allows for a preemptive "killer adoption" of the technology. If countries with strong currencies adopt the technology underlying cryptocurrencies through launching CBDC early enough, they can nip the future growth of cryptocurrencies in the bud. Otherwise it is only until the cryptocurrency market has gained widespread adoption that the implementation of CBDC becomes unavoidable. As a result, the stronger country's strategy on whether to launch CBDC evolves from an offensive, preemptive tactic to a purely defensive measure. Regardless, our model predicts that the digitization of money becomes inevitable in the long run.

An early launch of CBDC benefits countries with non-dominant currencies (country B) the

most, as long as their currencies are not too weak. The incentives of the relatively weaker country B to launch CBDC are stronger than A's incentives, and are primarily shaped by the desire to obtain a technological first-mover advantage from launching CBDC early on. Conversely, the dominance of the strongest currency A (e.g., U.S. dollar) causes "entrenchment" and lack of direct competition which limits the incentives of country A to implement CBDC. Overall, our analysis suggests a pecking order of CBDC development, with countries with strong but non-dominant currencies (e.g., China and South Korea) spearheading the endeavors, followed by countries with the strongest currencies (e.g., the United States), and then by nations with the weakest or non-existent sovereign currencies (e.g., El Salvador).[4] In other words, a country's incentive to develop CBDC follows an inverted U shape in the strength of its currency (relative to other currencies).

Decisions to launch CBDC can be either strategic substitutes or complements. Our model highlights that through the launch of CBDC, weaker currencies may challenge the dominance of stronger currencies. If it poses a threat on the dominance of the stronger currency, the implementation of CBDC by weaker countries increases the incentives of the stronger country to launch CBDC too, giving rise to strategic complementarity in CBDC issuance. Consistent with our model, China's e-CNY is often perceived as such a threat to the dominance of the U.S. dollar and, accordingly, has led calls to action (Ehrlich, 2020, Forbes) for the United States to consider the development of CBDC too. In contrast, CBDC issuance by stronger countries eliminates the possibility for weaker countries to attain a technological first-mover advantage, thereby always reducing weaker countries' incentives to develop CBDC and giving rise to strategic substitutability in CBDC issuance.

We further study the implications of CBDC issuance by A on developing countries with particularly weak currencies (bigger gap between A and B). Consistent with Brunnermeier et al. (2019), we find that such countries are particularly prone to digital dollarization: they tend to suffer the most when a country with strong currency implements CBDC. Yet, these developing countries and their currencies do not benefit much from implementing CBDC themselves, because their currency is weak regardless of its underlying technology. Our analysis suggests that developing countries may benefit from adopting cryptocurrency as a legal means of payment within their own territory instead of implementing CBDCs as a way to escape from (digital) dollarization.

The development of various digital currencies therefore can be viewed as financial innovation that eventually benefit households and businesses (e.g., Duffie et al., 2021). Our model can be used

[4]China's digitization of RMB is also believed to be driven by the need to compete with private payment platforms such as AliPay or Wechat Pay, which cause similar disruptions to the banking sector as cryptocurrencies.

to understand how currency competition and the strength of national currencies impact financial innovation. In particular, the weakness of national currencies implies a vacuum in the currency space which favors the emergence of (private) cryptocurrencies and thus financial innovation in the private sector. Moreover, as crytpocurrencies gain widespread adoption, countries' incentives to innovate through the implementation of CBDC increase too, further stimulating financial innovation. On the other hand, the dominance of national currencies curbs incentives for innovation both for governments and the private (financial) sector. In short, competition among national currencies as well as the rise of cryptocurrencies stimulate financial innovation.

Finally, our framework also applies to the study of fiat-backed cryptocurrencies, especially stablecoins (e.g., USDC) which are typically pegged to the U.S. dollar and (partially) backed by U.S. dollar assets such as physical dollars or Treasury bills. When a cryptocurrency is backed by reserves consisting of currency A, country A can capture part of the seigniorage generated from cryptocurrency usage, which strengthens currency A but weakens other currencies. These findings suggest that the United States and the U.S. dollar may benefit from regulation that requires stablecoin issuers to hold U.S. dollar reserves instead of regulation that restricts or bans stablecoin issuance. Furthermore, we find that the appropriate regulation of stablecoins (i.e., with regulatory reserve requirements) could allow countries with the strongest currencies, such as the United States, to effectively "delegate" the creation of a CBDC ("digital dollar") to the private sector, which is a viable alternative to developing an own CBDC to compete with cryptocurrencies.

Literature. Our paper adds to the literature on currency competition (see e.g., Lagos and Wright, 2005) or competition among reserve and safe assets (see e.g., He, Krishnamurthy, and Milbradt, 2019). Farhi and Maggiori (2018) develop a model of the international monetary system, and study competition among countries in the provision of reserve assets. Fernández-Villaverde and Sanches (2019) analyze competition between private cryptocurrencies and government money. Brunnermeier et al. (2019) argues that the digitization of money leads to unbundling and rebundling of the roles of money and fiercer competition of specialized currencies, which affects exchange rates and monetary policy (see e.g., Benigno, 2019). Most closely related to our paper is Benigno, Schilling, and Uhlig (2022) who develop a model of currency competition between two fiat currencies and one global cryptocurrency. Benigno et al. (2022) show that the presence of a global cryptocurrency (if used) synchronizes monetary policy across countries, giving rise to the classical Mundel-Flemming

trilemma or Impossible Trinity. Our analysis mainly differs from these papers as it (i) highlights the impacts of CBDC introduction on currency competition and price dynamics, (ii) studies countries' incentives to implement CBDC, and (iii) offers a game-theoretic analysis of currency digitization.

Our discussion on global digital currency competition is related to ongoing policy debates, regulatory hearings, industry initiatives, and CBDC (Bech and Garratt, 2017; Duffie and Gleeson, 2021; Duffie, 2021; Prasad, 2021; Giancarlo, 2021). In particular, our paper adds to the emerging literature on CBDCs and stablecoins. Bech and Garratt (2017), Auer and Böhme (2020); Auer, Frost, Gambacorta, Monnet, Rice, and Shin (2021), MAS (2021), Mancini-Griffoli, Peria, Agur, Ari, Kiff, Popescu, and Rochon (2018), Duffie et al. (2021) provide overviews and surveys about CBDCs. Many articles analyze the interaction between the banking sector and CBDCs (Fernández-Villaverde, Schilling, and Uhlig, 2020; Bindseil, 2020; Bordo and Levin, 2017; Davoodalhosseini, 2021; Brunnermeier and Niepelt, 2019; Piazzesi and Schneider, 2020; Parlour, Rajan, and Walden, 2020; Fernández-Villaverde, Sanches, Schilling, and Uhlig, 2021). In particular, several studies examine the impact of CBDCs on deposit and lending markets within a country, and its dependence on bank competition, market frictions, and design features (Chiu, Davoodalhosseini, Jiang, and Zhu, 2019; Andolfatto, 2021; Keister and Sanches, 2021; Garratt and Zhu, 2021). Ferrari, Mehl, and Stracca (2020) analyze open-economy implications of CBDCs.

More broadly, our study contributes to the recent literature on blockchain economics and cryptocurrencies.[5] Biais, Bisiere, Bouvard, Casamatta, and Menkveld (2018), Schilling and Uhlig (2019), Pagnotta (2021), Cong, Li, and Wang (2021b), and Sockin and Xiong (2021) provide theoretical foundations for token pricing while Hu, Parlour, and Rajan (2019), Liu and Tsyvinski (2021), Liu, Tsyvinski, and Wu (2019), Makarov and Schoar (2020), and Cong, Karolyi, Tang, and Zhao (2021) empirically document cryptocurrency return patterns. While a large part of the literature focuses on consensus generation and the design or functionality of tokens (e.g., Rogoff and You, 2019; Prat, Danos, and Marcassa, 2021; Cong and Xiao, 2021; Cong et al., 2021a; Garratt and Van Oordt, 2021; Gryglewicz, Mayer, and Morellec, 2021; Mayer, 2022; Prat and Walter, 2021; Sockin and Xiong, 2022), they do not examine the competition among various digital currencies, including the ones issued by central banks as well as private parties. Lyons and Viswanath-Natraj (2020), Kozhan and Viswanath-Natraj (2021), Gorton and Zhang (2021), Guennewig (2021), Rout-

[5] Chiu and Koeppl (2019), Cong and He (2019), and Easley, O'Hara, and Basu (2019) are among the earliest contributions. For a literature review on blockchain economics, see, e.g., Chen, Cong, and Xiao (2021), and John, O'Hara, and Saleh (2021).

ledge and Zetlin-Jones (2021), and Li and Mayer (2021) analyze stablecoins issued by private entities. Our paper adds to this literature as it studies competition among national fiat currencies, CBDCs, and private cryptocurrencies, notably, including stablecoins.

1 Dynamic Model of Currency Digitization and Competition

Time (indexed by t) is infinite without any discounting. To introduce households and money, we set up the model "as if" time runs discretely with time increments $dt > 0$, i.e., $t = 0, dt, 2dt, 3dt, ...$ We take the continuous time limit $dt \to 0$ once we complete the model description.[6]

Households and consumption. The economy is populated by one representative OLG household which takes prices as given. Cohort t is born at t with lifespan dt and exits at $t + dt$ when a new cohort is born. At birth, each cohort is endowed with one unit of the perishable generic consumption good which serves as the numeraire that all prices are quoted in. Cohort t derives utility from consumption only at time $t + dt$ and thus would like to store their endowment (consumption good) from t to $t + dt$, yet the consumption good cannot be stored. As a consequence, money — which comes in the form of currencies A, B, and C (discussed below) — serves as a store of value.

Currencies. Two countries, A and B, have their national (fiat) currencies A and B respectively. In addition, there is one representative cryptocurrency C. Each currency $x \in \{A, B, C\}$ is in fixed unit supply and has equilibrium value (i.e., price) P_t^x in consumption goods.[7] The three currencies A, B, and C fulfill the three functions of money, i.e., they potentially serve as (i) store of value, (ii) medium of exchange, and (iii) unit of account. We refer to the country with the stronger currency, which has higher value in consumption goods at time $t = 0$, as the "strong" country, and to the other country as the "weak" one. Without loss of generality, we set country A to be strong, and currency A can be viewed as reserve currency with $P_0^A \geq P_0^B$. One can think of the dominant currency A as the U.S. dollar, while B is a relatively weaker currency (e.g., Euro or RMB).

Importantly, the representative cryptocurrency describes the broader cryptocurrency market, including stablecoins which are cryptocurrencies pegged to a reference unit. Many of the largest stablecoins (e.g., USDC or BUSD) are pegged to the U.S. dollar and are (partially) backed by U.S.

[6] We model OLG households in a continuous time economy following He and Krishnamurthy (2013).
[7] Section 5.5 discusses currency supply changes and monetary neutrality in our framework.

dollar reserve assets including physical U.S. dollars or cash equivalents like U.S. Treasury bills. To capture that part of the cryptocurrency market value may be backed by U.S. dollar reserves, we stipulate that fraction $\theta \in [0, 1)$ of the crytocurrency market capitalization P_t^C is backed by currency A.[8] Any regulation that requires stablecoins to be backed to a larger extent by currency A would increase θ. Notice that $\theta P_t^C / P_t^A$ units of currency A are held in reserve out circulation and thus cannot be held by the household, thereby leaving $1 - \theta P_t^C / P_t^A$ units of currency A as circulating supply for the household. As will become clear later, our parametric assumptions ensure that $P_t^A > \theta P_t^C$, so that the fraction $\theta P_t^C / P_t^A$ is well-defined and lies between zero and one.

Money as a store of value and market clearing. To consume at time $t + dt$, cohort t uses their consumption good endowment to buy money from the previous cohort (i.e., cohort $t - dt$) at time t. At time $t + dt$, cohort t exchanges money for the consumption good with cohort $t + dt$ and so on. To initialize the model, we assume that the first cohort born at $t = 0$ buys currency $x = A, B$ from the central bank (government) of country x at prices P_0^A and P_0^B respectively as well as the cryptocurrency C from its developers at price P_0^C.

We denote by m_t^x cohort t's holdings of currency x in terms of the consumption good over their lifetime $[t, t + dt]$. As cohort t does not derive any utility from consuming early at time t, cohort t invests their entire endowment of one consumption good into money, which implies:

$$m_t^A + m_t^B + m_t^C = 1. \tag{1}$$

Notice that cohort t is the only holder of currencies B and C, so $m_t^B = P_t^B$ and $m_t^C = P_t^C$ by market clearing of currencies B and C. Currency A is both used as reserve asset backing cryptocurrency C and held by the household. The reserve backing cryptocurrency is worth θP_t^C units of the consumption good, while the household holds the remainder of currency A. By market clearing, $m_t^A = P_t^A - \theta P_t^C$. Observe that market clearing implies a direct link between currency usage and adoption, captured by m_t^x, and currency strength or value, captured by P_t^x. Next, the market clearing conditions for currencies and (1) imply

$$P_t^A + P_t^B + P_t^C(1 - \theta) = 1. \tag{2}$$

[8] While stablecoins could be also backed by fiat currencies other than the U.S. dollar, we emphasize that this is rarely the case in practice and that the fraction of cryptocurrency market capitalization backed by fiat currencies other than the U.S. dollar is negligible. We therefore consider that currency C is only backed by currency A.

For simplicity, at any time, the aggregate value of money in terms of the consumption good held by the household equals the endowment, i.e., the real value of the economy is fixed, and currency competition is a zero sum game in terms of the consumption good. One could easily introduce growth in the consumption goods.

Convenience and money as a medium of exchange. Money also serves as a medium of exchange, both across and within cohorts, or provides liquidity services to its holders as in Benigno et al. (2022). We account for these functions in reduced form by assuming that households derive a convenience yield from holding money, reminiscent of the money-in-the-utility-function approach frequently adopted in the classical monetary economics literature.[9] Formally, the lifetime utility of cohort t can be written as:

$$U_t = c_{t+dt} + Z_o[m_t^A + m_t^B + m_t^C]dt + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt, \tag{3}$$

where c_{t+dt} denotes cohort t's consumption at time $t + dt$ and the remainder terms capture the convenience yield of holding money over $[t, t + dt]$. Cohort t derives a constant (marginal) base convenience yield Z_o regardless of which currency she holds. The constant $Z_o \geq 0$ is chosen large enough to ensure that the convenience yield to holding currency x, that is, $Z_o m_t^x + Z_t^x v(m_t^x)$ for $x = A, B$ and $Z_o m_t^C + Y_t v(m_t^C)$ for $x = C$, is positive and is otherwise immaterial. The convenience yield cohort t derives from holding m_t^x numeraire units in currency x grows with Z_t^x for $x = A, B$ and Y_t for $x = C$ respectively, and, as in Krishnamurthy and Vissing-Jorgensen (2012), is further characterized by a concave function $v(m_t^x)$ that is twice differentiable satisfying $v'(m_t^x) > 0$ and $v''(m_t^x) < 0$. The parameters $Z_t^x \geq 0$ and $Y_t \geq 0$ may relate to the (payment) technology underlying currency $x = A, B$, and C respectively, and capture all differences in currencies' convenience yields. Alternatively, the convenience yield parameters may also reflect (unmodelled) interest rate differentials across different currencies or that certain currencies, such as the U.S. dollar, fulfill a demand for safe assets.[10] As we will see later, higher Z_t^x or Y_t stimulates usage and holdings of

[9] This money-in-utility approach follows studies on monetary economics (e.g., Sidrauski, 1967; Feenstra, 1986; Poterba and Rotemberg, 1986; Walsh, 2017). It is also related to the convenience yield of money-like securities such as treasuries (e.g., Krishnamurthy and Vissing-Jorgensen, 2012). The precise way in which money generates convenience (new monetarist approach) is not of particular importance for conveying our insights.

[10] For simplicity, we do not model monetary policy which may affect the currency interest rate differential and currencies' convenience yield. As we focus on the game-theoretic analysis of countries' strategies to digitize their currency, we leave this area for future research and refer readers who are interested in the role of monetary policy in currency competition between fiat and cryptocurrencies to Benigno (2019) and Benigno et al. (2022).

currency x. To ensure that equilibrium currency holdings are strictly positive, i.e., $m_t^x > 0$, and each currency x has positive value, we assume $\lim_{m_t^x \to 0} v'(m_t^x) m_t^x = \infty$. In particular, the marginal utility of holding an infinitesimal amount of currency x goes to infinity, reflecting imperfect currency substitutability (as, e.g., in Benigno et al., 2022).[11]

Inflation and currency strength. We consider that over $[t, t+dt)$, country x levies "inflation taxes" of $\tau_t^x dt$ units of the consumption good from its currency holders so as to cover its expenses, such as the costs of servicing debt, international trade expenses, or its fiscal deficit. In reduced form, τ_t^x is inversely related to the economic or fiscal strength of country x, whereby high (low) τ_t^x represents weak (strong) economic fundamentals and implies high (low) inflation. The assumption that country x levies inflation taxes which decrease with its economic/fiscal strength is a tractable way to model the empirically relevant (positive) link between a country's fiscal or economic strength and the strength of its currency (Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2020) relative to the numeraire or the reserve currency.[12]

In practice, countries with weaker currencies oftentimes borrow debt denominated in the reserve currency and international trade is mostly invoiced in the reserve currency (e.g., U.S. dollars), suggesting that the strength of the reserve currency affects countries' expenses and economic strength as well as the strength and inflation of their currency.[13] Motivated by these observations and to capture the role of the reserve currency (i.e. USD) as international unit of account, we assume that τ_t^x increases with the value or strength of the strong currency A, that is, P_t^A.[14]

Formally, over $[t, t+dt)$, country x raises $\pi^x dt$ units of currency A plus $\kappa^x dt$ units of the consumption good as taxes from the currency holders (i.e., cohort t), where $\kappa^x \geq 0$ and $\pi^x \geq 0$

[11] For instance, many transactions within a certain country have to be settled with the local currency; cryptocurrencies may provide usage not available through fiat currencies.

[12] One could also model this link between fiscal strength and currency strength by stipulating that the convenience yield of currency x (e.g., parameter Z_t^x) directly depends on the economic fundamentals of country x. Our results are robust as long as a country's fiscal strength improves the benefits of holding its currency.

[13] Du, Pflueger, and Schreger (2020) show that countries which are able to issue more domestic currency debt are also the ones that issue more debt denominated in foreign currency; Maggiori et al. (2020); Maggiori, Neiman, and Schreger (2019) document that U.S. dollar is the primary currency of denomination (over 60%) since the 2008 crisis in cross-border investors portfolio holdings, even when neither the investor nor the issuer are based in the United States; a dollar dominance similarly manifests in invoicing traded goods (e.g., Goldberg and Tille, 2008; Gopinath et al., 2020; Gopinath and Stein, 2021), consistent with the international use of the dollar as a unit of account (e.g., Matsuyama, Kiyotaki, and Matsui, 1993; Doepke and Schneider, 2017); Gourinchas (2019) and Jiang, Krishnamurthy, and Lustig (2020, 2021), among others, further elaborate on the dollar dominance.

[14] We assume that even if the value of currency B temporarily exceeds the value of currency A, currency A continues to serve as the global unit of account. This assumption reflects that the reserve currency/unit of account status is typically sticky and does not change with transitory value fluctuations (Gopinath et al., 2020).

are exogenous constants.[15] Expressed in terms of the consumption good, total taxes of country x are $\tau_t^x dt := (\kappa^x + \pi^x P_t^A)dt$. As a result, holding one unit of currency A over $[t, t+dt)$, one incurs a tax of $\tau_t^x dt$ units of the consumption good. Thus, holding one unit of the consumption good in currency x or, equivalently, $1/P_t^x$ units of currency x, one incurs taxes $(\tau_t^x/P_t^x)dt$ in terms of the consumption good over $[t, t+dt)$. These taxes can and will be interpreted as inflation, because country x could collect taxes of $\tau_t^x dt$ units of consumption good from cohort t by minting additional $\tau_t^x/P_t^x dt$ units of its currency x. Selling new units of its currency to cohort $t+dt$ at time $t+dt$, country x would cause the debasement of its currency x and inflation, diluting cohort t.[16] For simplicity, we do not explicitly model such currency supply changes and inflation which we discuss in more detail in Section 5.5. In particular, Section 5.5 argues that due to monetary neutrality, taxes τ_t^x could be transformed into currency depreciation of the same magnitude, and so are akin to inflation. Therefore, we refer to τ_t^x or τ_t^x/P_t^x (with some abuse of notation) as inflation rate.

Crucially, if the strong currency P_t^A appreciates, the inflation rate τ_t^B of currency B increases, so that currency B depreciates in terms of its consumption value. This mechanism is akin to "imported inflation" from which countries in real life may suffer when their currency depreciates relative to the U.S. dollar; that is, country B is exposed to some form of "dollarization." Taking stock, currency B features high inflation τ_t^B/P_t^B when its currency is weak relative to the consumption good (due to $\kappa^B \geq 0$) or relative to currency A (due to $\pi^B \geq 0$). High inflation compromises the store of value function of currency B and, as we will see, discourages usage of currency B.

Different to national currencies, cryptocurrency holdings are not subject to inflation tax, but, due to endogenous price dynamics, might be subject to inflation and depreciation relative to the consumption good or other currencies when P_t^C decreases over time.[17]

Cryptocurrencies. National currencies and cryptocurrencies differ in their convenience yield parameters Z_t^x and Y_t. In our model, one key difference between cryptocurrency and national currency is the dynamic adoption and growth of the cryptocurrencies. Specifically, the cryptocurrency

[15] A direct interpretation is that over $[t, t+dt)$, country x raises taxes from currency holders to cover expenses that are partially denominated in the consumption good and partially in the strong currency, which is the international unit of account. Also note that in principle, one could allow κ^x or π^x to take negative values, which does not qualitatively affect our results as those results are driven by the difference of τ_t^A and τ_t^B but not by the exact levels.

[16] For such inflation to be consistent with fixed unit supply of currency x, one could renormalize the supply of each currency at the beginning of each period to one.

[17] It would be straightforward to introduce a tax or cost of holding cryptocurrency, which would have similar effects as a reduction in the cryptocurrency convenience yield. For theoretical clarity, we omit such taxes or costs and may account by adjusting Y.

market and its underlying technology grow endogenously according to

$$\frac{dY_t}{Y_t} = \mu m_t^C dt, \tag{4}$$

where $\mu \geq 0$ to reflect that greater adoption m_t^x today spurs growth and innovation, i.e., network scale feeds back positively to the sector's growth. Cryptocurrency usage, as captured by m_t^C, stimulates the growth of the technology underlying cryptocurrencies and so financial innovation. Intuitively, a higher level of m_t^C motivates developers to improve the technology underlying cryptocurrencies and expand use cases. We assume that the potential convenience yield of cryptocurrencies is bounded, in that $Y_t \leq \overline{Y}$ for some exogenous constant $\overline{Y} > 0$.[18] Formally, the drift of dY_t vanishes as it reaches $\overline{Y}$ (i.e., $dY_t = 0$ if $Y_t = \overline{Y}$) while (4) holds for $Y_t < \overline{Y}$. We set $Y_0 \geq 0$.

In our framework, banning or regulating cryptocurrencies by any country (or both) can be interpreted as reducing usability and thus the convenience to holding cryptocurrencies, captured by Y. One can then reinterpret Y as the convenience net of the effect of regulation or a ban. Bans and regulations of cryptocurrencies might not always be feasible and may stifle innovation. Cryptocurrencies may also offer unique convenience and therefore compete with fiat money even when heavily regulated or banned. In light of this competition from the forefront of payment innovations, a country may respond by adopting technologies and digitizing its currency through the launch of their own digital currency, known as Central Bank Digital Currency (CBDC). As we focus on countries' strategic decisions to digitize their currency to compete with cryptocurrency or other countries' currencies, we do not explicitly model bans or regulation of cryptocurrency and leave this for future research.

Central Bank Digital Currency (CBDC). CBDCs are generally believed to have advantages over fiat currencies in, e.g., improving cross-border payments, lowering the cost of providing physical money, promoting financial inclusion, enabling smart contracting and programmable money, reducing depository counterparty risk, and help monetary policy implementation such as the dissemination of government relief payments during the pandemic (e.g., Foundation and Accenture, 2020; Duffie et al., 2021, Page 7).[19] We introduce CBDC in a technology-neutral fashion that does

[18] This assumption is inconsequential for our key findings and is only made for regularity purposes. In fact, one can take the limit $\overline{Y} \to \infty$.

[19] See also the witnessing for Economic Affairs Committee, House of Lords, UK Parliament (Duffie and Gleeson, 2021), October 12, 2021. We recognize that CBDC designs are work in progress and some of the advantages are a

not rely on any specific design, while acknowledging that there are many different designs and implementations as well as specific benefits of CBDC, the study of which is beyond the scope of the paper.[20] We merely interpret CBDC as a technological innovation which improves upon traditional fiat money and increases convenience yield of currency x. Formally, when a country $x = A, B$ launches CBDC at some time T^x, then the convenience yield parameter Z_t^x jumps up, in that

$$Z_t^x = \begin{cases} Z_L & \text{for} \quad t < T^x \\ Z_H + \alpha Y_t & \text{for} \quad t \geq T^x \end{cases},$$

where $\alpha \geq 0$ and $0 \leq Z_L \leq Z_H$ are positive constants. Z_t^x is public knowledge. Note that the gains of CBDC implementation partially depend on the the state of the cryptocurrency market and its underlying technology, which reflects the notion that the technology underlying CBDC to some extent derives from the technology underlying cryptocurrencies (e.g., blockchain technology and smart contracts).[21] Because the increase in convenience leads to an increase in relative demand of the currency as a store of value and unit of account, implementing CBDC naturally has an impact of a currency's other functionalities as well.

The implementation of CBDC constitutes a major disruption and requires the support from multiple parties and regulatory approval that all take time, effort, and investment.[22] To capture

promise but not a guarantee. Depending on the design, CBDCs have downsides such as breaking the complementarity of deposit and credit lines, exacerbating lending inequalities, or reducing deposits and investments (Piazzesi and Schneider, 2020; Parlour et al., 2020; Keister and Sanches, 2021), and the alteration of the informational environment through smart contracting and tokenization (Cong and He, 2019; Lee, Martin, and Townsend, 2021). Our specification captures the net benefits of the digitization of payment systems and currencies, which are well-recognized (e.g., Prasad, 2021). Notably, in a New York Times interview on February 22, 2021, Treasury Secretary Yellen remarked: "Too many Americans don't have access to easy payments systems and banking accounts, and I think this is something that a digital dollar, a central bank digital currency, could help with." Moreover, CBDCs are a source of profit and seigniorage revenue, but with reduced cost to taxpayers for production and for Anti-Money Laundering (AML) and tax collection; interest-paying CBDCs may also reduce intermediary rent to the banks. A retail CBDC would also preserve the relevance of generally-accessible central bank money in a digital economy, safeguarding consumer and merchant interests as commerce moves further online (MAS, 2021), as well as increasing interoperability across platforms to keep public money relevant (Brunnermeier et al., 2019).

[20] CBDCs could be directly managed by the central banks or indirectly managed through banks. Direct CBDCs are also divided into (deposit) account-based or token-based, with the former most closely resembling electronic payment systems such as PayPal or Alipay while the latter potentially involving both digital tokens issues by central banks and technology firms or conventional banks providing customers with synthetic CBDCs fully backed by segregated central bank deposits. Retail CBDCs also differs from wholesale CBDCs. Given the large literature studying these issues, we model CBDCs as technology-neutral and are agnostic of the (technical) details on the design and implementation.

[21] Arguably, the introduction of CBDCs could spur the development of the cryptocurrency sector too. But at the moment no CBDC is directly used on private or decentralized blockchain platforms. We therefore do not model such a spillover, which intuitively would mitigate the impact of CBDC implementation on cryptocurrencies.

[22] For example, many see support from the banking sector as vital to the success of a digital U.S. dollar, however commercial banks in the U.S. have taken a largely adversarial stance. According to Duffie (2021), "the development

this feature, we assume that the (random) time T^x at which country x successfully launches CBDC arrives according to a jump process $dJ_t^x \in \{0,1\}$, with intensity λe_t^x. Here, $\lambda \geq 0$ and $e_t^x \geq 0$ is the endogenous effort or, in other words, investment of country x to implement CBDC. That is, $\mathbb{E}_t[dJ_t^x] = \lambda e_t^x dt$, and $T^x = \inf\{t \geq 0 : dJ_t^x = 1\}$. Effort e_t^x is costly and entails private flow costs or disutility $\frac{(e_t^x)^2}{2}$ for country x. The costs $\frac{(e_t^x)^2}{2}$ take the form of disutility, do not affect inflation taxes τ_t^x, and are denominated in consumption goods. Effort e_t^x is not contractible and is not publicly observable (i.e., effort e_t^x is only observed by country x).

Government objective function. At any time t, the government or central bank of country $x = A, B$ chooses its effort (taking the effort of the other country as given) so as to maximize the following objective:

$$V_t^x = \max_{(e_s^x)_{s \geq t}} \mathbb{E}_t^x \left[\int_t^\infty e^{-\delta(s-t)} \left(\delta g_s^x(m_s, P_s) - \frac{(e_s^x)^2}{2} \right) ds \right], \tag{5}$$

where $\mathbb{E}_t^x[\cdot]$ denotes the time-t expecation from the perspective of country x (which is conditional on time-t public information and effort $(e_s^x)_{s \geq t}$). In (5), $g_s^x(m_s, P_s)$ is a function that may depend on the full vector of time-s equilibrium currency adoption levels $m_s := (m_s^A, m_s^B, m_s^C)$ and currency values $P_s := (P_s^A, P_s^B, P_s^C)$. In what follows, we take $g_s^x = \beta P_s^x$ for a constant $\beta \geq 0$, so that the government would like to maximize (a weighted time average of) total usage, adoption, strength, or value of its own currency, all captured by P_t^x, which is akin to maximizing seigniorage or total currency value. The government objective from (5) may also capture economic, fiscal, or geopolitical considerations, in as much these are reflected in the usage, adoption, strength, or value of currency x (which all are quantified by P_t^x). Also note that we model a (reduced form) link between a country's economic/fiscal strength and the strength of its currency via the taxes τ_t^x: thus, a country's economic/fiscal strength — ceteris paribus — relates positively to the strength, value, or adoption of its currency.[23]

Notably, $\delta \geq 0$ captures how much the government cares about current usage of its currency versus future usage. In the limit $\delta \to \infty$, the government becomes effectively myopic and chooses

of an effective and secure digital dollar will require significant resources and time, perhaps more than five years."

[23] To motivate a positive link between currency usage and value and a country's economic fundamentals, one may also invoke the quantity theory of money which states that — holding money supply and velocity of money fixed — a higher level of economic activities and transactions (which may be due to stronger economic fundamentals) with currency x boosts demand, value, and usage of currency x.

e_t^x to maximize the expected change in currency usage less the disutility of effort (or the cost of investment), i.e., the time-t objective becomes $\mathbb{E}_t^x[dP_t^x] - \frac{(e_t^x)^2}{2}dt$. When $\delta \to \infty$, the government cares about the future only in as much prices incorporate future information. Section 5.4 provides some additional discussion of the government's objective, and studies comparative statics in δ.

2 Model Solution

We now characterize the dynamic equilibrium in the continuous time limit $dt \to 0$. To begin with, let the state variable $z \in \{0, A, B, AB\}$ denote which countries have launched CBDC up to date. Specifically, $z = 0$ means that no country has launched CBDC yet, $z = A$ means that only country A has launched CBDC, $z = B$ means that only country B has launched CBDC, and $z = AB$ means that both countries A and B have launched CBDC. We focus on a Markov equilibrium with state variables (Y, z), so that all equilibrium quantities can be expressed as functions of (Y, z). In equilibrium, at any time $t \geq 0$, cohort t chooses the holdings of currencies A, B, C to maximize the expected utility $\mathbb{E}[U_t]$ (with U_t from (3)), given prices (P_t^A, P_t^B, P_t^C). The markets for all currencies clear, i.e., $m_t^A = P_t^A - \theta P_t^C$, $m_t^B = P_t^B$, and $m_t^C = P_t^C$. And, both countries A and B choose their efforts according to (5).

To solve for the Markov equilibrium, we first define the expected returns of currency x in terms of the consumption good as

$$r_t^x = \frac{\mathbb{E}_t[dP_t^x]}{P_t^x dt}, \tag{6}$$

where $\mathbb{E}_t[\cdot]$ denotes the time-t expectation which is conditional on all publicly information that is available at time t. Notice that r_t^x is the expected rate of appreciation of currency x in terms of consumption good. That is, if $r_t^x > 0$, currency x is expected to appreciate, causing deflationary pressure in terms of the consumption, and, if $r_t^x < 0$, currency x is expected to depreciate, causing inflationary pressure. In equilibrium, r_t^x is a function of (Y, z), i.e., $r_t^x = r^x(Y, z)$.

Next, we can write cohort t's consumption c_{t+dt} at $t + dt$ as:

$$c_{t+dt} = \sum_{x \in \{A,B,C\}} \frac{m_t^x P_{t+dt}^x}{P_t^x} - \sum_{x \in \{A,B\}} \frac{\tau_t^x m_t^x}{P_t^x} dt. \tag{7}$$

Basically, cohort t's consumption consists of the proceeds from selling their nominal holdings of currency x, m_t^x/P_t^x, at price P_{t+dt}^x to cohort $t+dt$ minus the taxes cohort t pays to countries A and

B. As argued above, these taxes τ_t^x can be viewed as inflation. The interpretation is that country x could collect taxes by printing/selling more money and keeping the proceeds from doing so, while the households bear the costs of this inflation. Taxes are a deadweight loss for the household.

We can write $P_{t+dt}^x = P_t^x + dP_t^x$ and, inserting this relation into (7), we obtain:

$$c_{t+dt} = \sum_{x\in\{A,B,C\}} m_t^x + \sum_{x\in\{A,B,C\}} \frac{m_t^x dP_t^x}{P_t^x} - \sum_{x\in\{A,B\}} \frac{\tau_t^x m_t^x}{P_t^x} dt. \tag{8}$$

Because cohort t only derives utility from consuming at time $t+dt$, it is optimal to use the entire endowment one to purchase money at time t, so that $\sum_{x\in\{A,B,C\}} m_t^x = 1$ must hold for given prices (P_t^A, P_t^B, P_t^C) (see (1)). As a result, cohort t maximizes

$$\max_{m_t^A, m_t^B, m_t^C \geq 0} \mathbb{E}_t[U_t] \quad \text{s.t.} \quad \sum_{x\in\{A,B,C\}} m_t^x = 1, \tag{9}$$

taking (P_t^A, P_t^B, P_t^C) as given. With (3), (8), and $\sum_{x\in\{A,B,C\}} m_t^x = 1$, the objective in (9) becomes:

$$\mathbb{E}_t[U_t] = 1 + \sum_{x\in\{A,B,C\}} m_t^x r_t^x dt - \sum_{x\in\{A,B\}} \frac{\tau_t^x m_t^x}{P_t^x} dt + Z_o dt + Z_t^A v(m_t^A) dt + Z_t^B v(m_t^B) dt + Y_t v(m_t^C) dt.$$

The first three terms represent the expected consumption of cohort t at time $t+dt$, which is the unit endowment plus the expected returns to investing in currencies A, B, and C, less the taxes levied by countries A and B. The last four terms represent the convenience yield to holding currencies.

In light of $\sum_{x\in\{A,B,C\}} m_t^x = 1$, it must be in optimum that

$$\frac{\partial \mathbb{E}_t[U_t]}{\partial m_t^A} = \frac{\partial \mathbb{E}_t[U_t]}{\partial m_t^B} = \frac{\partial \mathbb{E}_t[U_t]}{\partial m_t^C}, \tag{10}$$

provided $m_t^x \in (0,1)$. That is, in equilibrium, the household is on the margin indifferent between substituting a unit of currency x towards another currency $-x$. As stated in Proposition 1 below, this relationship implies the following equilibrium pricing equations:

$$Y_t v'(m_t^C) + r_t^C = Z_t^A v'(m_t^A) + r_t^A - \frac{\tau_t^A}{P_t^A} = Z_t^B v'(m_t^B) + r_t^B - \frac{\tau_t^B}{P_t^B}. \tag{11}$$

Condition (11) states that in equilibrium, the sum of the marginal convenience yield to holding cryptocurrencies and expected cryptocurrency returns equals the sum of the marginal convenience

yield to holding national currency x and its returns net the inflation currency x carries due to taxation. Due to $\lim_{m_t^x \to 0} v'(m_t^x) m_t^x = \infty$, the currency holdings m_t^x and values P_t^x that solve (11) satisfy $P_t^x \in (0,1)$ for $x = A, B, C$. In a Markov equilibrium with state variables (Y, z), we can write $m_t^x = m^x(Y,z)$ and $P_t^x = P^x(Y,z)$ for $x = A, B, C$ as well as $r_t^x = r^x(Y,z)$ so that (11) will depend on (Y,z) only.

Interestingly, (11) reflects feedback effects in currency competition. A decrease in demand for and value P_t^B of currency B increases the inflation τ_t^B / P_t^B currency B faces, which in turn discourages households to hold currency B and reduces the value of currency B further. Notably, this effect is amplified because due to market clearing (2), a decrease in currency value P_t^B generally implies an increase in currency value P_t^A which further exacerbates inflation of currency B. Consequently, currency usage and dominance exhibit strong network effects, and causality runs both ways.[24] The dominant currency A has less inflation/depreciation than B because it is the more valuable currency; A is the stronger currency because it has less inflation/depreciation.

Next, we characterize government's time-t value function from (5) as well as the optimal levels of efforts. By the dynamic programming principle, the governments' value function V_t^x from (5) satisfies the HJB equation:

$$\delta V_t^x = \max_{e_t^x \geq 0} \left(\beta \delta P_t^x - \frac{(e_t^x)^2}{2} + \frac{\mathbb{E}_t^x[dV_t^x]}{dt} \right). \tag{12}$$

Again, in a Markov equilibrium with state variables (Y,z), we can express V_t^x as a function of (Y,z) only, in that $V_t^x = V^x(Y,z)$ for $x = A, B$. As optimal effort e_t^x is determined according to the HJB equation (12), it depends on the government's value function $V_t^x = V^x(Y,z)$ and currency values $P_t^x = P^x(Y,z)$ as well as changes therein. Since V_t^x and P_t^x are functions of (Y,z) only, optimal effort is a function of (Y,z) too, in that $e_t^x = e^x(Y,z)$. As we show in the Appendix A, optimal effort satisfies for $x = A, B$ (where $x = A$ implies $-x = B$ and vice versa):

$$\begin{aligned} e^x(Y,0) &= \lambda(V^x(Y,x) - V^x(Y,0)) \\ e^x(Y,-x) &= \lambda(V^x(Y,AB) - V^x(Y,-x)) \\ e^x(Y,AB) &= e^x(Y,x) = 0. \end{aligned} \tag{13}$$

[24] In fact, a similar force could be obtained by modelling network effects in reduced form (e.g., Cong, Li, and Wang, 2021c; Cong et al., 2021a). Also note that in practice, speculation to inflation could even exacerbate the "vicious circle of inflation" (Obstfeld and Rogoff (1983)).

We summarize our findings in the following Proposition.

Proposition 1. *In a Markov equilibrium with state variables (Y, z), the following holds:*

1. *Households invest their entire endowment in currencies, i.e., (1) holds. The markets for all currencies clear, so that $m_t^A = P_t^A - \theta P_t^C$, $m_t^B = P_t^B$, $m_t^C = P_t^C$. As a result, condition (2) holds.*

2. *The equilibrium pricing condition (11) holds, and government value functions V_t^A and V_t^B solve the HJB equation (12).*

3. *For $x = A, B, C$, currency value satisfies $P_t^x = P^x(Y_t, z_t)$, currency usage satisfies $m_t^x = P^x(Y_t, z_t)$, expected currency returns (in terms of the consumption good) satisfy $r_t^x = r^x(Y_t, z_t)$, government value function satisfies $V_t^x = V^x(Y, z)$, and optimal effort satisfies $e_t^x = e^x(Y, z)$, whereby optimal effort is characterized in (13).*

4. *The Markov equilibrium is characterized by a system of coupled first order ODEs which is presented in Appendix A.5.*

In the proof of the proposition in Appendix A, we provide the detailed characterization of the model solution in terms of a system of coupled ODEs that describe the dynamics of the currency values $P^x(Y, z)$, currency usage $m^x(Y, z)$, and governments' value functions $V^A(Y, z)$ and $V^B(Y, z)$ as well as effort $e^A(Y, z)$ and $e^B(Y, z)$. The system of ODEs can then be solved numerically.

3 Simplified Framework

We start by studying a simplified version of the model which allows to derive some of our findings in analytical form and delivers some intuition before proceeding to the fully dynamic model. Specifically, we consider a "static version" of the model in which the state variables (Y, z) remain constant over time, i.e., we assume $\lambda = 0$ and $\mu = 0$ so that $r_t^x = 0$ and $e_t^x = 0$. We write $P^x := P_t^x$, $m^x := m_t^x$, and $Z^x := Z_t^x$. Internet Appendix C presents a static model variant with two time periods including its detailed analysis, and shows that the results of the static model are similar to the ones from the benchmark of this Section with constant (Y, z).

Throughout, we follow Li (2021) to specify the convenience yield in the CRRA functional form:

$$v(m_t^x) = \frac{(m_t^x)^{1-\eta} - 1}{1 - \eta}. \tag{14}$$

For simplicity, we set $\kappa^A = \kappa^B = \theta = 0$, and take $\eta = 2$. We present our results as comparative statics in the "adjusted (marginal) convenience yield" of cryptocurrency, denoted $\hat{Y} := Yv'(m^C)$ which we treat (with abuse of notation) as a parameter. The following Proposition demonstrates that $\hat{Y}$ quantifies cryptocurrency adoption (i.e., P^C increases with $\hat{Y}$) and illustrates the effects of the rise of cryptocurrencies, captured by an increase in $\hat{Y}$.

Proposition 2. *Cryptocurrency value P^C increases with $\hat{Y}$. The rise of cryptocurrencies harms the strong currency A, i.e., P^A decreases with $\hat{Y}$. But, the rise of cryptocurrencies may benefit the weak currency B: If and only if $\pi^B > \sqrt{2}\pi^A$, there exists an interval $[0, \underline{Y})$ with $\underline{Y} > 0$ on which P^B increases with $\hat{Y}$. For sufficiently large $\hat{Y}$, P^B decreases with $\hat{Y}$.*

Proposition 2 yields the following insight.

Insight 1: Cryptocurrencies harm strong currency A but may benefit the weaker currency B. Note that the cryptocurrency value P^C increases with $\hat{Y}$, implying that $\hat{Y}$ quantifies cryptocurrency adoption and the size and value of the cryptocurrency market/sector. The rise of cryptocurrencies unambiguously harms the strong or reserve currency A, in that P^A decreases with $\hat{Y}$. The cryptocurrency growth reduces the demand for both currency A and B, thereby decreasing P^A and P^B. However, as currency A depreciates, country B's expenses denominated in currency A fall too, which reduces inflation and benefits currency B. The rise of cryptocurrency weakens currency B as a direct competition but at the same time reduces the degree of competition currency B faces from currency A. When the strong currency is sufficiently dominant and π^B is sufficiently large (i.e., $\pi^B > \sqrt{2}\pi^A$), this second effect dominates at low values of $\hat{Y}$. Put differently, the cryptocurrency market acts as a "buffer zone" amidst the competition of national currencies A and B, weakening the feedback between currency usage and inflation/depreciation.

Countries may react to growing competition from cryptocurrencies as well as other national currencies by digitizing their currency through the launch of CBDC. In our model, the implementation of CBDC by country x is akin to an increase in the convenience yield of currency x.[25] Suppose a country cares about its seigniorage revenue through maximizing its currency value P^x. Then, $\frac{\partial P^x}{\partial Z^x}$ measures a country's incentives to launch CBDC or simply the effects of CBDC issuance on currency value, even though we formally shut down countries' effort to implement CBDC. The

[25] This increase is net of potential privacy costs, which constitute an important research topic in its own (e.g., Liu, Sockin, and Xiong, 2020).

following Proposition presents some comparative statics in Z^x.

Proposition 3. *Cryptocurrency value decreases with Z^x for $x = A, B$. Suppose that $\hat{Y} = 0$. Then:*

$$sign\left(\frac{\partial P^A}{\partial Z^A} - \frac{\partial P^B}{\partial Z^B}\right) = sign(\pi^B - 2\pi^A). \tag{15}$$

Thus, when $\pi^B \in (\pi^A, 2\pi^A)$ and $\hat{Y} \geq 0$ is sufficiently low, country B benefits more from issuing CBDC than the strong country does, in that $\frac{\partial P^A}{\partial Z^A} < \frac{\partial P^B}{\partial Z^B}$. In addition, $\frac{\partial P^C}{\partial Z^B} = -\frac{\partial P^B}{\partial Z^B} < -\frac{\partial P^A}{\partial Z^A} \leq \frac{\partial P^C}{\partial Z^A}$ for $\pi^B < 2\pi^A$ and $\hat{Y}$ sufficiently low.

According to Proposition 3, CBDC issuance, as captured in reduced form by an (exogenous) increase in Z^x, offers the largest advantages for countries with non-dominant but relatively strong currencies (characterized by relatively low π^B), such as China or strong emerging economies like India. These countries should also have the strongest incentives to launch CBDC, which is consistent with the first large scale CBDC launch by China and not the United States.[26] We gain the following insights.

Insight 2: Country B's CBDC poses a greater threat to cryptocurrencies. Given $\frac{\partial P^C}{\partial Z^B} = -\frac{\partial P^B}{\partial Z^B} < -\frac{\partial P^A}{\partial Z^A} \leq \frac{\partial P^C}{\partial Z^A}$ for $\pi^B < 2\pi^A$ and sufficiently low $\hat{Y} \geq 0$, our findings also suggest that CBDC issuance by countries with strong but non-dominant currencies like China or India pose a bigger threat to cryptocurrencies than CBDC issuance by the United States does. The intuition is that cryptocurrencies mainly compete with weaker currencies rather than the reserve currency, so that any appreciation by weaker currencies harms the cryptocurrency market value more.

Insight 3: Pecking order of CBDC issuance. Overall, we observe a pecking order of CBDC issuance. Countries with non-dominant but relatively strong currencies, such as China or India, benefit the most from implementing CBDC, followed by countries such as the United States that are already dominant in the global currency competition. Countries with very weak currencies (e.g., $\pi^B > 2\pi^A$), such as El Salvador, benefit the least from CBDC issuance, because $\frac{\partial P^B}{\partial Z^B}$ decreases with π^B. Intuitively, the currency of these countries is weak regardless of the implementation of CBDC,

[26] The key motivations of China for introducing eCNY are cited as limiting the dominance of private payment services. However, both mobile service provision and eCNY, once more international, can challenge U.S. dollars and Euros. After all, eCNY technology likely opens commercial opportunities for China in some emerging markets, amplifying China's influence in emerging economies, something U.S. and EU foreign policy experts may have to consider.

and CBDC issuance by such countries has negligible impact on the strong country's currency or the cryptocurrency market. These countries may find it advantageous to directly adopt non-pegged cryptocurrencies as legal means of payment within their territory.

4 Model Implications and Discussion

Notably, the battle of currencies, the rise of cryptocurrencies, and countries' strategic decisions to launch CBDC are inherently dynamic; the static benchmark with $\mu = \lambda = 0$ consequently cannot shed light on these issues. We therefore gain additional insights by analyzing the fully dynamic model. As such, we numerically solve the fully dynamic model with $\mu > 0$ and $\lambda > 0$ characterized by the system of ODEs from the Appendix A.5. We then derive predictions on how the rise of cryptocurrencies shapes currency competition and shape incentives of various governments to launch their own digital currencies.

For the numerical solution, we assume that, as in the previous Section 3, $v(m^x)$ takes the CRRA functional form in (14). To afford maximal theoretical clarity, we aim to reduce the number of free parameters (which have to be chosen), and therefore normalize $\kappa^A = \kappa^B = \theta = 0$ as well as $\mu = \lambda = \beta = 1$, which is akin to removing these parameters from the model. Section 5.2 studies comparative statics in θ. Thus, the differences of countries A and B are captured entirely by the differences in π^A and π^B, whereby π^x inversely captures country x's economic or fiscal strength which then affects the strength of its currency.[27] Further, we normalize $Z_L = 1$, $\delta = 1$, and $\pi_A = 1$. We set $Z_H = 4$, $\pi^B = 4$, $\alpha = 0.15$, $\eta = 2$, and $\overline{Y} = 75$. The parameter Z_o does not determine currency holdings m_t^x or values P_t^x and thus can be set to an arbitrary value, for instance, such that the convenience yield to holding currency x is positive in all states. We initialize the model at $Y_0 = 0.01$ and $z = 0$, and over time the growth of Y_t is endogenously determined according to (4). The numerical solution yields an interior equilibrium, featuring $m_t^x = P_t^x \in (0, 1)$ in all states (at all times) and for all $x = A, B, C$. Importantly, the model's qualitative implications are robust to the choice of these parameters.

Our baseline specification considers π^A and π^B that are not too divergent, which describes the competition between fiats of major nations or regions, such as the U.S. dollar and the Euro or the

[27] Setting $\kappa^A = 0$ and $\kappa^B = 0$ is equivalent to eliminating these parameters from the model. We do so for the sake of theoretical clarity, but note that the model's qualitative implications are robust to the choice of κ^A and κ^B; i.e., our main findings would also arise, if we considered different values of κ^A and κ^B.

U.S. dollar and the Chinese Yuan. Meanwhile, country B could also be interpreted as a relatively strong emerging economy like India. Developing countries or weak emerging economies are heavily dependent on dollar financing (Du et al., 2020) and so are characterized by a large value of π^B. Our model is consistent with real-life observations that when π^B is sufficiently big, the presence of the weak country B has little influence on currencies A or C. More importantly, we study the incentives and effects of CBDC issuance by such countries in Section 5.3 where we formally consider the case of large π^B (with big gap from π^A).

Finally, note that in the numerical solution under our baseline parameters, we obtain that $P_0^A > P_0^B$. And, as time t approaches infinity, any country x has launched CBDC eventually so that $T^x < \infty$. And, once all countries have launched CBDC, the currency A dominates currency B, so that $\lim_{t\to\infty} P_t^A > \lim_{t\to\infty} P_t^B$. Thus, even if currency B is temporarily stronger than currency A, the "initial order of dominance" will be restored eventually, suggesting that currency A can be viewed as reserve currency irrespective of temporary fluctuations in currency values.

4.1 The Rise of Cryptocurrencies and Model Dynamics

We start by discussing the currency value and adoption dynamics. Figure 1 displays currency values P^x both as a function of $\ln(Y)$ (which is a monotonic transformation of Y) and calendar time t before any CBDC is launched ($z = 0$). Note that Y_t increases over time, and the rate of increase endogenously depends on cryptocurrency adoption and thus the cryptocurrency value. The solid black line depicts the baseline scenario $\pi^B = 4$ implying relatively strong currency B. The dotted red line depicts a scenario with a higher value of $\pi^B = 20$ implying relatively weak currency B.

Panels A and D display the value of currency A for different values of Y (or equivalently $\ln(Y)$) and t. The rise of cryptocurrencies unambiguously hurts the strong currency A, in that the value of currency A decreases with Y and over time t. Meanwhile, the cryptocurrency value in panels C and F increases with Y and over time t. Notice that before reaching the upper bound $\overline{Y}$, the growth of the cryptocurrency market is effectively exponential and P_t^C is increasing and convex in time t, which reflects dynamic network and feedback effects: Higher cryptocurrency usage and adoption at time t contributes to the growth in the underlying technology Y_t and boosts cryptocurrency adoption in the future. Panels B and E display the value of currency B for different values of Y (or equivalently $\ln(Y)$) and t. When π^B is low and currency B is relatively strong, the rise of cryptocurrency hurts currency B, as the value of currency B decreases with Y and over time.

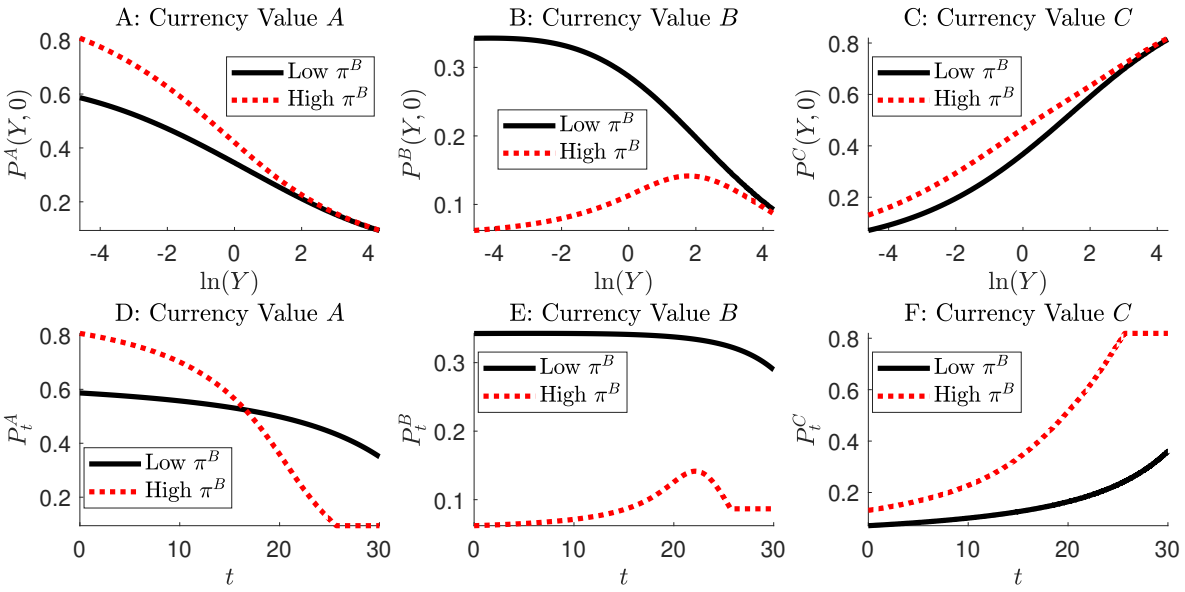

Figure 1: Model Dynamics. This figure plots currency values (or likewise usage and adoption) as a function of $\ln Y$ (upper panels A, B, and C) and a function of time t (lower panels D, E, and F) in state $z = 0$.

Interestingly, when π^B is high, the value of currency B is hump-shaped in Y and over time, i.e., first increases and then decreases with Y and over time. Then, the weaker country initially benefits from the rise of cryptocurrency. However, as the cryptocurrency market grows sufficiently large, it eventually limits usage of currency B, thereby damaging its value. The reason is that a stronger cryptocurrency market (i.e., an increase in Y) has two opposing effects on currency B. First, an increase in Y exacerbates direct competition currency B faces from cryptocurrencies, which makes households partially substitute their holdings of currency B for cryptocurrencies. Second, an increase in Y weakens currency A and therefore alleviates competition currency B faces from currency A. Weaker currency A reduces the inflation rate τ_t^B of currency B, which encourages households to hold more currency B. The first effect dominates for large values of Y while the second dominates for small values, leading to the aforementioned hump-shaped pattern of country B's currency value in $\ln(Y)$ and t. That is, the rise of cryptocurrencies may benefit sufficiently weak currencies, but harms stronger currencies.

Crucially, the endogenous growth of cryptocurrencies depends on the strength of the national currencies. For instance, when both currencies A and B are relatively strong and so have low inflation rates, the household's incentives to hold cryptocurrency are low too. By (4), low cryp-

tocurrency usage and adoption today stifles the growth of the crypto economy and the underlying technology, therefore implying low cryptocurrency usage and adoption in the future. In other words, the presence of strong national currencies hampers the emergence of privately-issued (crypto-) currencies. In contrast, a vacuum generated by weak national currencies, which prevails, e.g., when π^B is large, favors the emergence of cyptocurrencies, thereby spurring the growth of the cryptocurrency market and boosting the competition national currencies face from cryptocurrencies in the longer run.

Consequently, the strong country may actually benefit in the longer run from a stronger competitor B which is characterized by a lower value of π^B, in that the value P_t^A may increase in π^B at later times (see lower left panel). The reason is that when π^B is low, country A faces fierce competition from country B ex ante, but a strong currency B limits the growth of the cryptocurrency market and so limits competition from cryptocurrencies in the longer run. Conversely, when π^B is high, there is relatively low competition for currency A from currency B. However, the weakness of currency B encourages the rise of cryptocurrencies as competitor to currency A in the longer run.

4.2 The Effects of CBDC Implementation on Currency Values

We now study how the launch of CBDC by either country affects currency competition and pricing. Figure 2 plots the change in currency x's value if country x launches CBDC in state $z = 0$ (Panel A), the change in currency x's value if the other country (i.e., country $-x$) launches CBDC (Panel B), and the change in cryptocurrency value both in absolute (Panel C) and percentage terms (Panel D) when country x launches CBDC. The solid black line refers to currency $x = A$, and the dotted red line refers to currency $x = B$.

Panel A shows that upon the implementation of its own CBDC, the weak country's currency appreciates more (in absolute terms) than the strong country's currency. This suggests that the implementation of CBDC offers greater advantages for weaker currencies than for stronger ones. Panel B in Figure 2 depicts the effects of CBDC issuance by one country on the other's currency. Notice that currency A is harmed more by CBDC issuance of country B than currency B is harmed by CBDC issuance by country A. In other words, the strong currency suffers more from the CBDC implementation by its competitor than the weak currency.

Panels C and D in Figure 2 plot the change in cryptocurrency value when country A launches CBDC (solid black line) and country B launches CBDC (dotted red line) both in absolute terms

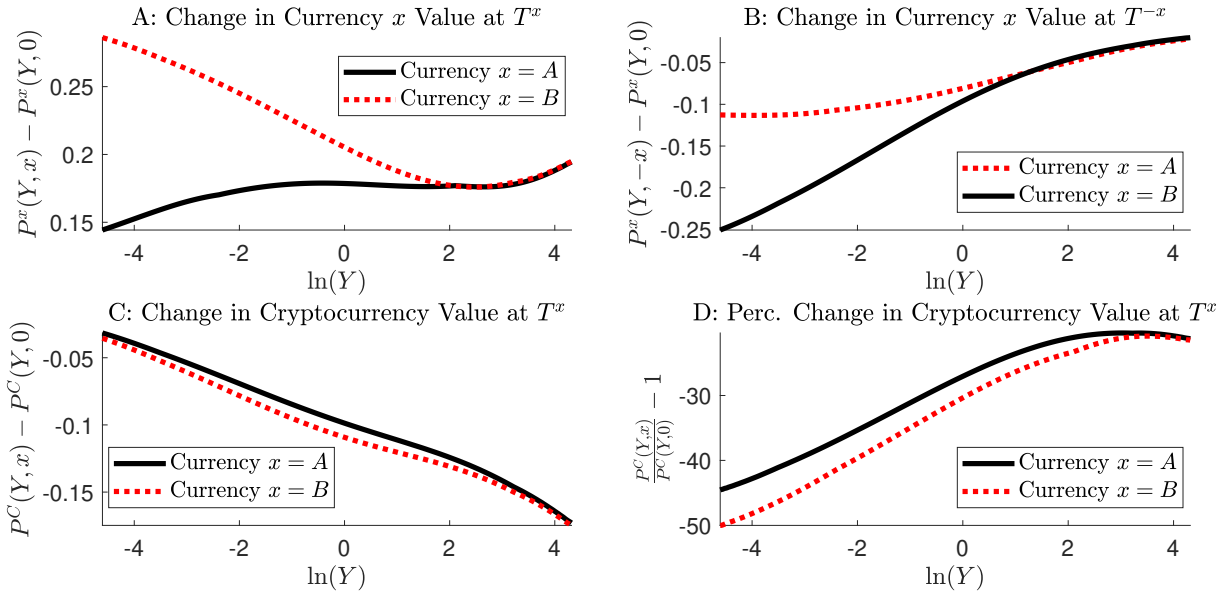

Figure 2: The effects of CBDC issuance on currency competition. This figure plots the changes in currency value when a country x successfully implement CBDC at time T^x in state $z = 0$.

(lower left panel) and percentage terms (lower right panels). Provided currency B is sufficiently valuable (i.e., π^B is not too large), CBDC issuance by the weak country has a more negative impact on the cryptocurrency value than CBDC issuance by the strong country. The intuition underlying this result is that cryptocurrencies mainly compete with currencies of relatively weaker countries rather than the reserve currency.

We conclude that (i) relatively strong but non-dominant currencies (such as the Euro or the Chinese Yuan) benefit more from CBDC issuance, (ii) dominant currency values (such as the U.S. dollar) tend to suffer more from competitor CBDCs, and (iii) the cryptocurrency market suffers the most when countries (currency unions) with relatively strong but not the dominant nations/regions, (e.g., China, India, or the Euro zone) implement CBDCs. According to our model, the implementations of CBDC by these countries pose more threat to the cryptocurrency market than the launch of CBDC by the dominant currency country, i.e., the United States.

4.3 The Incentives to Implement CBDC

Having studied the ex-post effects of CBDC issuance in state $z = 0$, we now characterize country x's incentives to launch CBDC as captured in $e_t^x = e^x(Y, z)$ determined according to (5). Crucially,

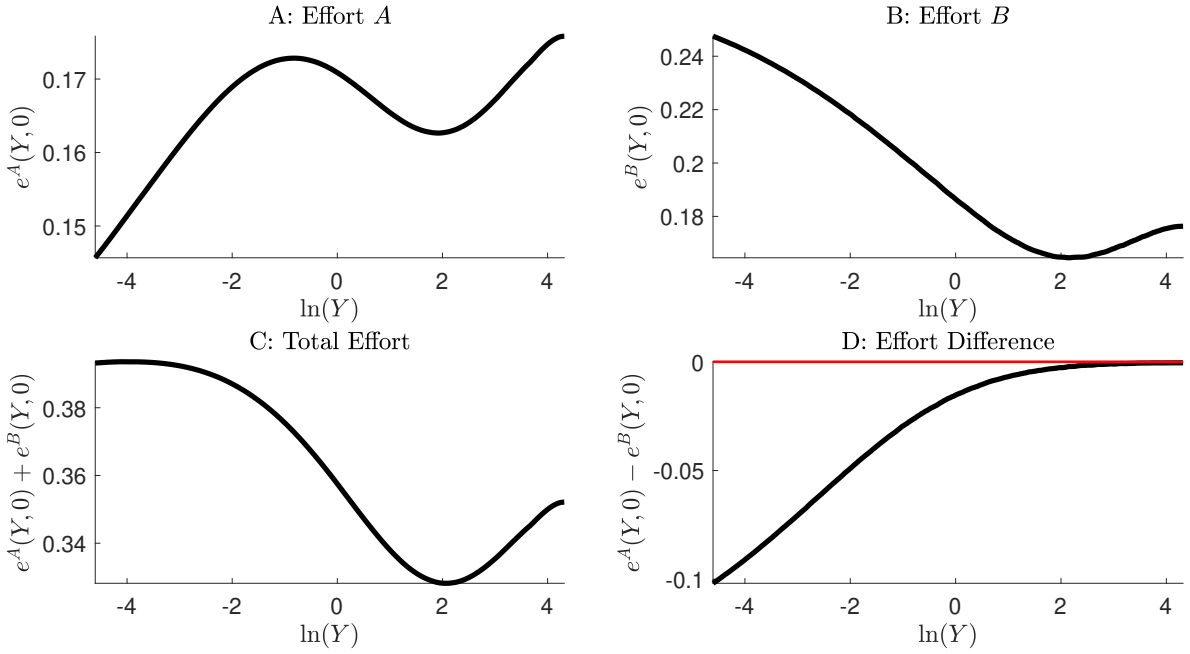

Figure 3: Countries' incentives and optimal efforts to launch CBDC. This figure plots countries' optimal efforts $e^x(Y,0)$ to implement CBDC (Panels A and B) in state $z=0$, as well as their sum (Panel C) and difference (Panel D).

these incentives depend on the size of the cryptocurrency market (as captured by Y) as well as on whether the other country has already launched CBDC (i.e., the state z). Importantly, the objective in (5) suggests that country x has high-powered incentives to launch CBDC if the contemporaneous currency value is low or the future (expected) currency value after launching CBDC is high. Both the currency value prior and after the launch of CBDC reflect the prevailing levels of currency competition and so do the countries' incentives to implement CBDC.

Note that when $z=0$, the incentives to implement CBDC reflect both a need to counteract the rising competition from cryptocurrencies and the prospect of attaining a technological edge over other national currencies. That is, the implementation of CBDC not only allows a country to compete more effectively with cryptocurrency but also gives the country an edge over the country which has not launched CBDC yet. This first-mover advantage lasts for a while after the successful launch of CBDC because CBDC implementation takes time and thus the other country cannot react immediately. In states $z=A$ and $z=B$, one country has attained such a first-mover advantage and no longer exerts effort. The other country consequently launches CBDC both to compete with the cryptocurrency market and to catch up to the other country in terms of technology.

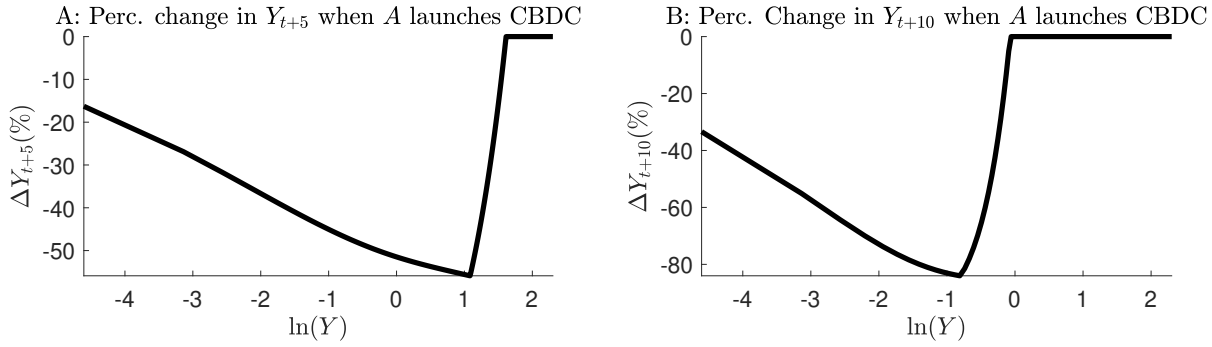

Figure 4: Crypto kill zone. This figure depicts a measure of the persistence of CBDC issuance by the strong country A in state $z = 0$. The two kinks reflect that Y_t or Y_{t+5} and Y_{t+10} hit the boundary $\overline{Y}$.

Figure 3 displays the efforts (incentives) of both countries (Panels A and B) as well as their differences and sums (Panels C and D) for different levels of Y in state $z = 0$, i.e., when no country has implemented CBDC yet. We start by discussing the strong country's incentives to launch CBDC in the upper right panel in Figure 3. Note that the strong country's effort is initially low when there is little competition from cryptocurrencies, in which case $P^A(Y, 0)$ is large and so the incentives to launch CBD are limited. In other words, the initial dominance of currency A reduces country A's incentives to innovate through developing CBDC. Over time, the cryptocurrency market rises as a competitor, thereby weakening currency A. As Y and cryptocurrency adoption increase, $P^A(Y, 0)$ decreases and, in turn, the incentives to launch CBDC ramp up. The competition from cryptocurrencies essentially incentivizes country A to adopt CBDC.

Because the cryptocurrency market's growth rate depends on the level of adoption m_t^C (see (4)), any reduction in m_t^C has persistent negative impact on future cryptocurrency adoption and value. Note that if country A launches CBDC relatively early (i.e., for low values of Y), the implementation of CBDC causes a significant reduction in future cryptocurrency adoption and value m_t^C. As a result, the launch of CBDC in the early stages of cryptocurrency adoption effectively "kills" the cryptocurrency market, hampering cryptocurrency adoption in the longer run. The possibility to cut down the cryptocurrency market in its early stages incentivizes country A to launch CBDC early on. In turn, the strong country's incentives to launch CBDC reach a peak in the so-called kill zone characterized by low values of Y where CBDC implementation by the strong country cuts down the cryptocurrecy market and, again, nips its growth in the bud.

Figure 4 provides an illustration. To understand this figure, consider two scenarios at t with $Y_t = Y$: (i) country A launches CBDC and (ii) country A does not launch CBDC. Figure 4 plots the percentage change in Y at time $t+5$ (Panel A) and time $t+10$ (Panel B) when country A launches CBDC at time t and state $z = A$ prevails until time $t+5$ and $t+10$ respectively as opposed to the scenario that no country launches CBDC and state $z = 0$ prevails until time $t+5$ and $t+10$ respectively. According to the figure, if country A launches CBDC early enough, it can achieve a significant (percentage) reduction in future cryptocurrency convenience or technology Y_{t+5} and Y_{t+10}. In contrast, if Y_t exceeds a critical threshold, CBDC issuance at time t no longer reduces the value of Y_t at future times $t+5$ and $t+10$. In other words, if country A launches CBDC early on, the effects of CBDC issuance on cryptocurrency adoption and Y are persistent.[28]

Loosely speaking, when the cryptocurrency market has grown sufficiently large and has reached a sufficient level of adoption, it is no longer possible to stifle its growth through the launch of CBDC, which reduces the benefits of launching CBDC. Thus, after the initial peak, country A's incentives to launch CBDC decrease again. Eventually, for sufficiently large values of Y, it becomes unavoidable to launch CBDC as a defensive measure to avoid full dominance of cryptocurrency. This leads to a double-peaked incentives to launch CBDC by the strong country as the crypto sector grows. That is, the strong country's strategy for launching CBDC evolves from an offensive, preemptive tactic to a purely defensive measure.

Intriguingly, as seen in Panel B in Figure 3, the weak country has high-powered incentives to launch CBDC early on, so as to attain a first-mover advantage in terms of technology and to reduce the degree of dollarization and competition from currency A. Note that competition from currency A is particularly strong for low values of Y, when the cryptocurrency market is in its infancy and currency A is strong. As the cryptocurrency market grows, currency A depreciates and so do the degree of dollarization and competition currency B faces from currency A. Consequently, country B's incentives to launch CBDC, which stem mainly from the desire to obtain a competitive advantage over currency A, taper off over time with the rise of cryptocurrencies. Importantly, we also find that the weak country's incentives to launch CBDC exceed the ones of the strong country (see Panel D in Figure 3), with difference in incentives tapering off for larger values of Y. Again,

[28] Admittedly, without further assumptions, the probability that Y_t reaches $\overline{Y}$ in the long run (i.e., as $t \to \infty$) is one. However, one could introduce a negative component to the drift of dY_t, say $\frac{dY_t}{Y_t} = \mu m_t^C dt - \delta dt$, in which case a reduction in m_t^C could imply for the long-run $Y_t \to 0$ instead of $Y_t \to \overline{Y}$. For simplicity, we do not formally introduce this effect.

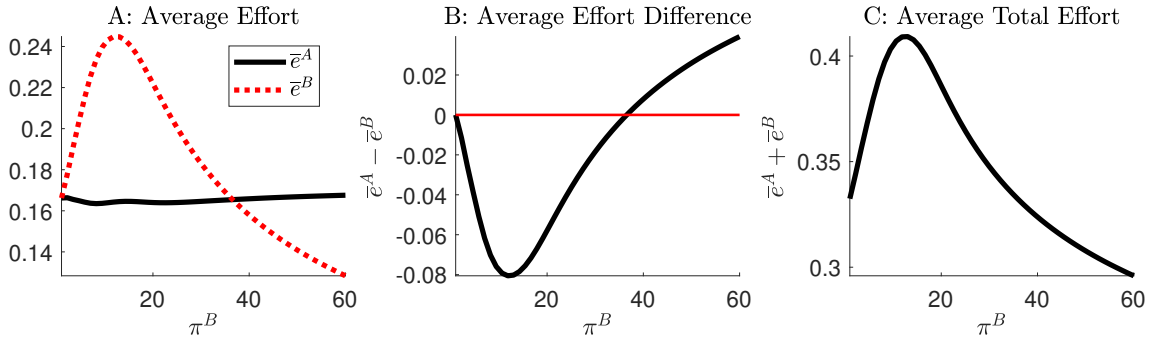

Figure 5: CBDC Pecking Order. This figure depicts countries' "average" efforts to implement CBDC $\overline{e}^x$ in state $z = 0$, and their relation to currency strength and competition.

these high-powered incentives of country B to implement CBDC reflect the competitive pressure currency B faces from currency A as well as the benefits of the potential technological first-mover advantage that B can attain by launching CBDC.

Last, Panel C in Figure 3 illustrates that countries' joint incentives to launch CBDC, $e^A + e^B$, tend to be highest for low values of Y. As such, our results suggest that the recent hype about CBDC issuances might be transitory and may taper off over time, as the cryptocurrency market expands further. However, eventually the (national) digitization of money is inevitable, in that joint effort to launch CBDC increases again for larger values of $\ln(Y)$.

4.4 The CBDC Pecking Order

Panel D in Figure 3 depicts the difference between country A's and B's incentives to launch CBDC in state $z = 0$ for different levels of Y under the parameter assumption $\pi^B = 4$, implying that currency B is relatively strong. Notice that because $e^A(Y,0) - e^B(Y,0) \leq 0$, country B has stronger incentives to launch CBDC first than country A, so as to gain a first-mover advantage. This finding suggests that countries with non-dominant but relatively strong currencies have particularly strong incentives to launch CBDC first, and these incentives exceed the ones of the countries with the strongest currencies.

To further study a country's incentives to launch CBDC and their relation to currency strength, Figure 5 plots a measure of country x's "average" effort in state $z = 0$, i.e.,

$$\overline{e}^x := \int_{Y_0}^{\overline{Y}} e^x(Y,0) d\ln(Y)$$

for different values of π^B, which captures the strength of currency B. Panel A shows that while A's average effort to implement CBDC $\bar{e}^A$ is relatively insensitive to changes in π^B, country B's average effort $\bar{e}^B$ is hump-shaped in π^B and thus highest for intermediate levels of π^B. This result predicts that countries with moderately strong currencies, which are in our model characterized by intermediate levels of π^B, have the strongest incentives to implement CBDC early on. Notably, these incentives exceed the ones of countries with the strongest or dominant currencies, i.e., country A or country B characterized by a low value of π^B. Last, countries with sufficiently weak currencies (i.e., countries with high π^B) have the lowest incentives to implement CBDC, as their currency is weak regardless. Average total effort in Panel C is also highest for intermediate levels of π^B, which is mainly driven by B's average effort being hump-shaped in π^B.

Taken together, our findings suggests the following pecking order for implementing CBDCs. First, countries with relatively strong but non-dominant currencies, such as China, the UK, and India, have the highest incentives to launch CBDC, and so are likely to launch (large-scale) CBDC first. Second, countries with the strongest or dominant currencies, e.g., the United States, are the next in line in terms of incentives and so are likely to implement CBDC after the countries with strong but non-dominant currencies. Third, countries with very weak currencies, characterized by a very large value of π^B, have negligible advantages from launching CBDC, since their currency is weak regardless. Put differently, a country's incentives to develop CBDC follow an inverted U shape in the strength of its currency (relative to other currencies).

4.5 Strategic Effects of CBDC issuance

The decision on whether to implement CBDC is strategic and crucially depends on whether other countries have launched CBDC. As discussed above, when $z = 0$, countries' incentives to implement CBDC reflect the hope to attain a technological first-mover advantage over the other country; when $z = A, B$, they reflect the need to catch up with the other country. We now study how country x's effort changes when the other country launches CBDC.

Figure 6 shows the percentage change in country A's effort when country B launches CBDC (Panel A) and the percentage changes in country B's effort when country A launches CBDC (Panel B) for different values of $\ln(Y)$. CBDC implementation by the strong country always reduces the weak country's incentives to implement CBDC, in that $\frac{e^B(Y,A)}{e^B(Y,0} - 1$ is negative. The reason is that for the weak country the main motive to launch CBDC is to gain a first-mover advantage over

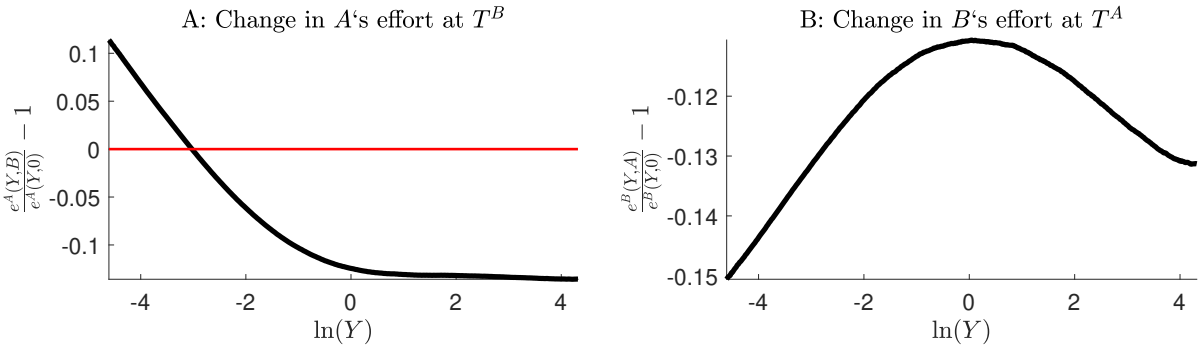

Figure 6: Strategic effects of CBDC issuance. This figure plots country x's effort change when the other country launches CBDC (at time T^{-x}).

currency A in technology. However, once A launches CBDC, it is no longer possible to gain this first-mover advantage.

Next, CBDC issuance by weak countries may increase or decrease the strong country's CBDC implementation effort. The intuition is that when Y is low and the value of currency A is big (Figure 1), CBDC issuance by the weak country causes drastic reduction in the value and dominance of currency A (Figure 2). In turn, the strong country would like to launch CBDC as well to defend or restore the dominance of its currency, which leads to this strategic complementarity.[29] Consistent with our model results, to the extent that the issuance of CBDC by China can be seen as such a threat to the dominance or reserve currency status of the U.S. dollar, it has led calls to action (Ehrlich, 2020, in Forbes) for America to consider the development of CBDC more seriously too. The recent hearing on stablecoins (United States Senate Committee on Banking and Affairs, 2021) and the fact the President Biden has recently signed an executive order on digital currencies constitute salient examples.[30] That said, the incentive is still smaller than country B's, as Duffie (2021) aptly puts, "Much has been written about the potential impact of eCNY, China's new CBDC, on the international dominance of the U.S. dollar. Concerns that the renminbi will rival the dollar in international markets are not warranted at this time, and these concerns are not a good reason to rush out a digital dollar before it is carefully designed."

[29] Outside the scope of our paper, it may advantage the United States to develop CBDC technology to offer the technology to countries that wish to lower the costs or advance the development time for introducing their own CBDCs (see, e.g., Duffie, 2021).

[30] See Fact Sheet March 09, 2022 from Statements and Releases, the White House.

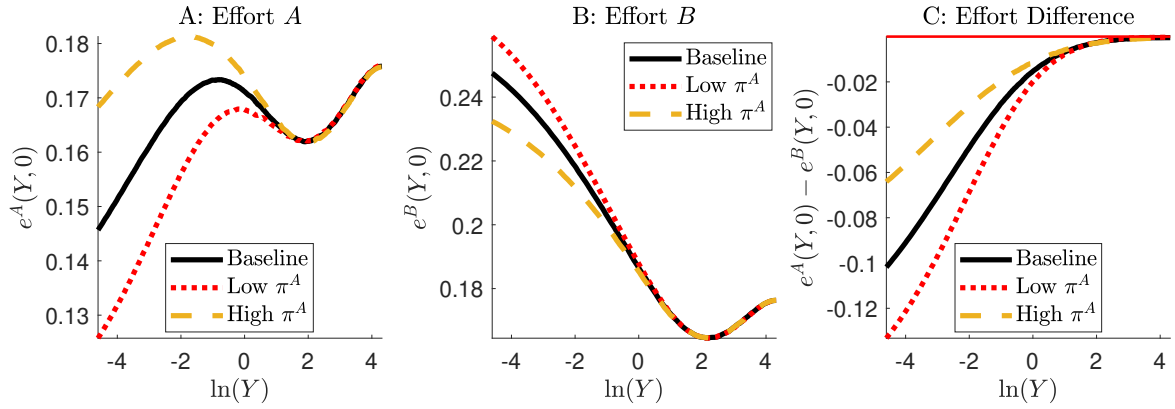

Figure 7: Fiscal strength and the incentives to launch CBDC. This figure plots countries' optimal effort $e^x(Y,0)$ to implement CBDC in state $z = 0$ (Panels A and B), as well as their differences (Panel C). We use our baseline parameters (i.e., $\pi^A = 1$). For the low value of π^A, we pick $\pi^A = 0$, and for the high value of π^A, we pick $\pi^A = 2$.

4.6 Currency Dominance and CBDC Issuance

We interpret currency A as the global reserve currency which in practice maps to the U.S. dollar. An increase in π^A means that the economic fundamentals of country A worsen, which feeds back into inflation and the currency value, undermining the dominance of currency A. Similarly, a decrease in π^A can be interpreted as a positive shock to economic fundamentals or as a negative shock to core inflation, reinforcing the dominance of currency A. We now study how a more dominant currency A affects countries' incentives to launch CBDC. Figure 7 plots the incentives of country A and B to launch CBDC against $\ln Y$ under our baseline parameters (solid black line; $\pi^A = 1$), for a lower value of π^A (dotted red line; $\pi^A = 0$), and for a higher value of π^A (dashed yellow line; $\pi^A = 2$).

As shown in Panel A in Figure 7, for any value of $\ln(Y)$, a stronger currency A (due to lower π^A) weakens country A's incentives to innovate by launching CBDC. These effects are amplified through the endogenous channel of the cryptocurrency market growth: Stronger currency A reduces cryptocurrency adoption and growth, which implies less competition for national currencies in the longer run and so undermines incentives to launch CBDC further. Panel B suggests that an increase in the the dominance of currency A (i.e., decrease in π^A) boosts B's efforts to implement CBDC. Intuitively, a more dominant currency A puts pressure on countries with relatively strong but non-dominant currencies, thereby incentivizing them to develops CBDC. Finally, Panel C shows that across different parameter values π^A, country B has stronger incentives to implement CBDC than

country A does.

Our analysis implies that a more dominant dollar makes the U.S. government less likely to implement CBDC, but stimulates other countries' efforts to implement CBDC. Conversely, weaker fundamentals, higher inflation, and thus more fierce competition among national currencies increases the incentives to implement CBDC, ceteris paribus. In as much the high core inflation in the United States (Santilli and Guilford, 2021) challenge the predominance of the dollar, this high inflation can also increase the government's incentives to accelerate dollar digitization, as seen in the recent release of discussion papers by the Federal Reserve Board or the executive order signed by President Biden.

5 Additional Results

5.1 Financial Innovation

Both the rise of cryptocurrencies and the implementation of CBDCs can be considered financial innovations that improve financial services and eventually benefit consumers (Duffie, 2021). We now study the determinants of this financial innovation. Recall that the endowment in our economy is fixed to one unit of the consumption "per period dt." Financial innovation thus only matters for the convenience yield households derive from holding currency. We consider two different measures of financial innovation: (i) Y_t which can be viewed as the technology underlying cryptocurrencies as a payment system; (ii) countries' propensity to innovate their currency by implementing CBDC, as quantified by the probability $Prob_t$ that at least one country has launched CBDC up to time t. In essence, Y_t measures financial innovation originating in the private (financial) sector, and $Prob_t$ measures government-induced financial innovation through CBDC development.

To examine how the strength of national currencies, quantified by π^A and π^B, relates to financial innovation through the emergence of cryptocurrencies and the implementation of CBDC, Figure 8 plots $Prob_t$ (see Panel A) and Y_t (see Panel B) against time t in the baseline (i.e., $(\pi^A, \pi^B) = (1, 4)$; solid black line), for relatively strong national currencies (i.e., $(\pi^A, \pi^B) = (0, 1)$; dotted red line), and for relatively weak national currencies ($(\pi^A, \pi^B) = (3, 10)$; dashed yellow line).[31] Note that for any t, both measures of financial innovation are higher when national currencies are

[31] To ensure some comparability, we maintain that $\pi^B = 3\pi^A + 1$, i.e., π^B is a linear increasing transformation of π^A with $\pi^B > \pi^A$. As such, an increase in π^A and π^B preserves to some extent the degree of competition between A and B, but weakens both national currencies relative to cryptocurrency.

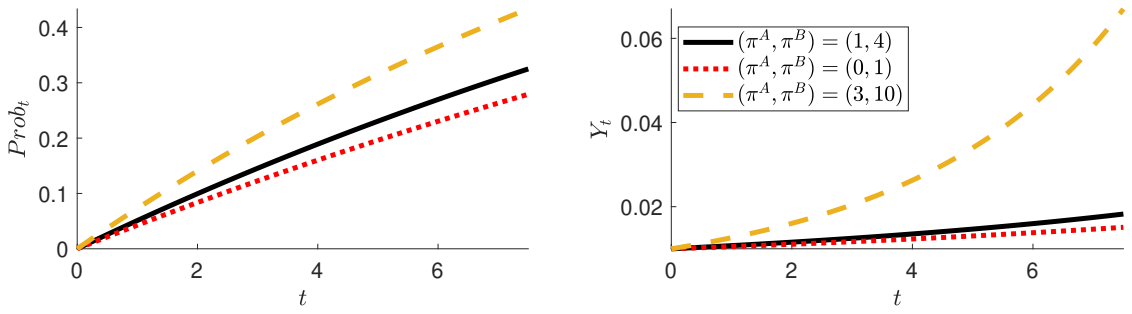

Figure 8: Financial Innovation. We use our baseline parameters (i.e., $\pi^A = 1$). This figure plots two measures of financial innovation against time, t. We use our baseline parameters. For the low value of π^A, we pick $\pi^A = 0$, and for the high value of π^A, we pick $\pi^A = 2$.

weaker, i.e., when (π^A, π^B) is larger. The intuition behind this finding is as follows. Relatively weak national currencies imply a vacuum in the currency space that is filled by crypocurrencies. High cryptocurrency adoption stimulates the growth of their underlying technology Y, encouraging financial innovation. And, the growth of cryptocurrencies feeds back into countries' decisions to innovate and eventually provides countries with high-powered incentives to launch CBDC, further increasing the degree of financial innovation.

Taking stock, weaker national currencies spur the rise of cryptocurrencies and countries' incentives to digitize their currencies through the implementation of CBDC, thereby stimulating financial innovation. In contrast, the dominance of national currencies stifles financial innovation and countries' incentives to launch CBDC. These results also suggest that the recent rise in core inflation in the US and in other developed economies might contribute to the growth of the cryptocurrency market, which in turn spurs potentially valuable financial innovation.

5.2 Stablecoins and U.S. Dollar Backed Cryptocurrencies

The representative cryptocurrency C describes the cryptocurrency market, including stablecoins. Many stablecoins (e.g., USDC or BUSD) are pegged to the U.S. dollar and partially backed by U.S. dollar assets, including physical dollars or cash equivalents like Treasury bills. In our model, the parameter θ captures the fraction of total cryptocurrency market value P_t^C which is backed by reserves consisting of currency A. Thus, an increase in θ could capture regulatory reserve requirements on stablecoins that require stablecoins to be backed to a larger extent by U.S. dollars (or other money-like claims such as Treasury bills). Likewise, the growing importance of stablecoins

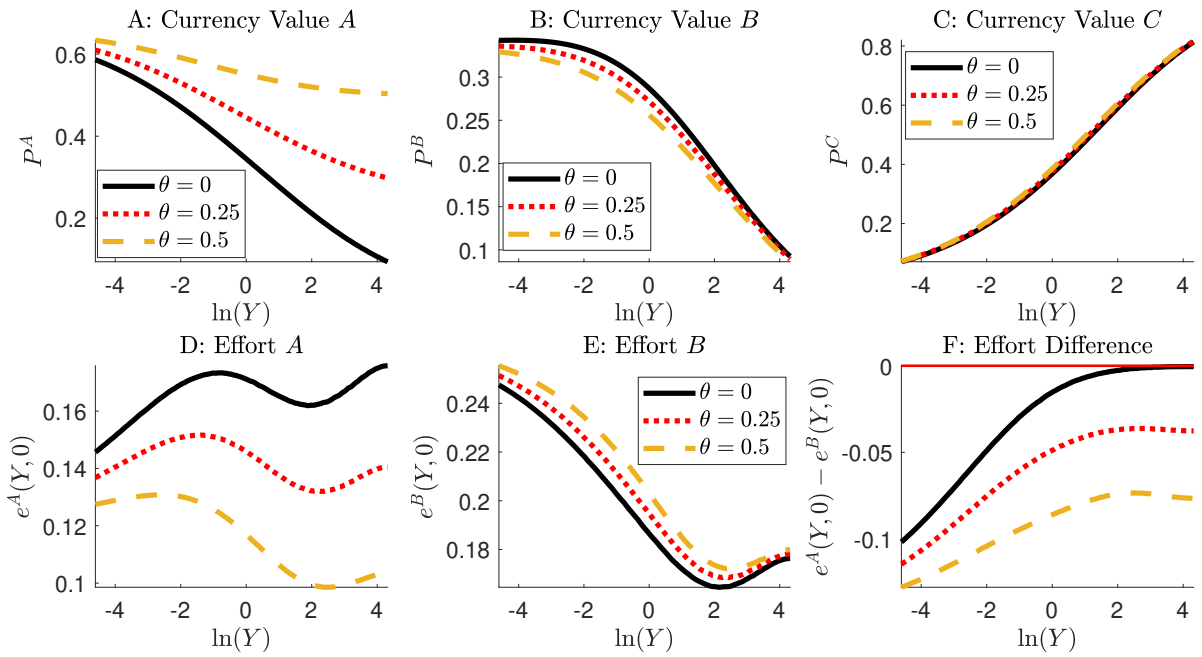

Figure 9: The effects of reserve-backed stablecoins. This figure presents comparative statics with respect to the parameter θ. Panels A, B, and C plots currency values against $\ln(Y)$ for different values of θ. Panels D, E, and F plot optimal effort $e^x(Y, 0)$ against $\ln(Y)$ in state $z = 0$ for different levels of θ.

(both within the cryptocurrency ecosystem or globally) could also trigger an increase in θ, so that an overall larger fraction of cryptocurrency value is backed by U.S. dollars.

We now analyze the effects of θ in our model, which yields some insights on the effects of U.S. dollar backed stablecoins and their potential regulation. Figure 9 plots currency values (see Panels A, B, and C) and efforts as well as their differences (see Panels D, E, and F) against $\ln(Y)$ for different values of θ. As can be seen from Panel A, an increase in θ unambiguously benefits currency A. All else equal, an increase in θ boosts demand for currency A as reserve asset for stablecoins, thereby raising A's value. Panels B and C show that an increase θ marginally reduces the value of currency B, but has only little effect on cryptocurrency value and adoption $P^C(Y, 0)$.

Interestingly, according to Panels D and E, a larger value of θ also undermines country A's incentives to implement CBDC, in that $e^A(Y, 0)$ decreases with θ for all values of $\ln(Y)$, but raises B's incentives to implement CBDC, in that $e^B(Y, 0)$ increases with θ for all values of $\ln(Y)$. The intuition is as follows. An increase in θ mitigates the adverse effects that cryptocurrencies have on the dominant currency A, thereby weakening the competition A faces from cryptocurrencies and so A's incentives to innovate by launching CBDC. Country B's incentives to implement CBDC increase with θ, because an increase in θ raises the value and adoption of currency A, which puts pressure on B to launch CBDC to counteract the competition from A.

These results suggest that requiring stablecoins pegged to the U.S. dollar to be backed by U.S. dollar assets can strengthen the dominance of the U.S. dollar, while weakening other national currencies. When stablecoins are backed by U.S. dollar assets, part of the seigniorage created by the cryptocurrency and stablecoin issuance in particular accrue to the United States. Facilitating regulated issuance of U.S. dollar stablecoins, the U.S. could "delegate" the creation of a digital dollar to the private sector, whilst capturing part of the generated seigniorage revenues. That is, U.S. dollar stablecoins can effectively export a digital version of the U.S. dollar to other countries or the digital economy in which cryptocurrency is adopted, possibly increasing the "reach" and global influence (and exorbitant privilege) of the U.S. dollar. Indeed, our model shows that A's optimal effort to implement CBDC decreases with θ, which suggests that the implementation of CBDC and regulatory reserve requirements on stablecoins, causing an increase in θ, are substitutes in currency digitization.

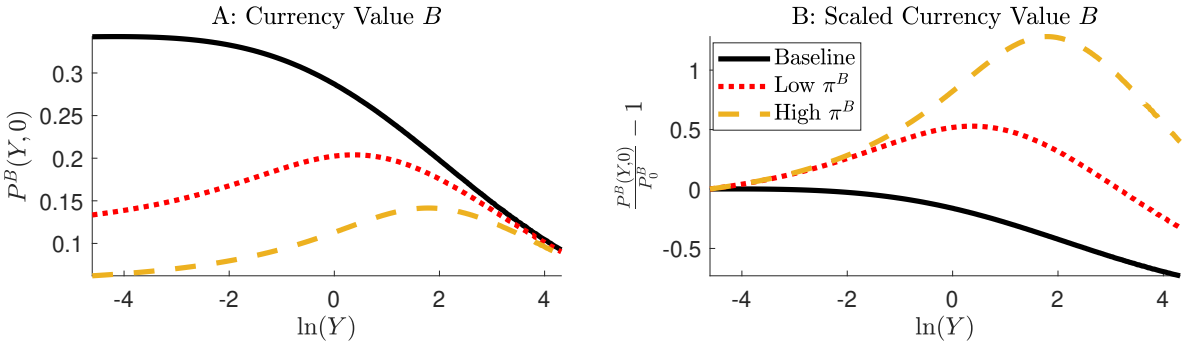

Figure 10: Which countries benefit from and adopt cryptocurrencies? This figure depicts how the rise of cryptocurrencies (i.e., an increase in Y) affects differentially currencies with different π^B. We use our baseline parameters (i.e., $\pi^B = 4$). For the low value of π^B, we pick $\pi^B = 2$, and for the high value of π^B, we pick $\pi^B = 20$.

5.3 Developing Countries and Digital Currencies

In this Section, we study the setting in which country B is characterized by a very large value of π^B, which would be the case for countries like El Salvador or Venezuela. In our model, a bigger π^B corresponds to higher inflation, weaker economic fundamentals, and a weaker currency B. A first observation is that $\lim_{\pi^B \to \infty} P_t^B = 0 \ \forall \ t \geq 0$. As such, $\lim_{\pi^B \to \infty} \mathbb{E}[dP_t^B]/dt = 0$, which, by (5), implies that countries with sufficiently high inflation rates and weak currencies do not benefit from implementing CBDC. Intuitively, the currency of a developing country is weak regardless of its underlying technology, which mechanically limits the gains from launching CBDC.

Our analysis suggests that these countries tend to benefit the most from adopting cryptocurrency as a legal means of domestic payment. As shown in Figure 1, weak countries may benefit from the rise of cryptocurrencies, in that P^C increases with Y for low values of Y. However, the extent of the benefit crucially depends on its fundamentals. We argue that developing countries characterized by large values of π^B are more likely to benefit from the rise of cryptocurrencies.

To formalize this argument, Panel A in Figure 10 plots the value of currency B against $\ln(Y)$, which quantifies technology, size, and adoption of cryptocurrencies for different values of π^B. Notice that an increase in Y unambiguously harms currency B when π^B is low, but may benefit currency B for larger levels of π^B. Interestingly, the higher π^B, the more currency B benefits from the rise of cryptocurrencies, in that $P^B(Y,0)$ reaches its peak for a larger value of Y.[32] Panel B plots

[32] More formally, the value of Y maximizing the value of currency B, which is the peak of $P^B(Y,0)$ in Figure 10, is larger for higher values of π^B.

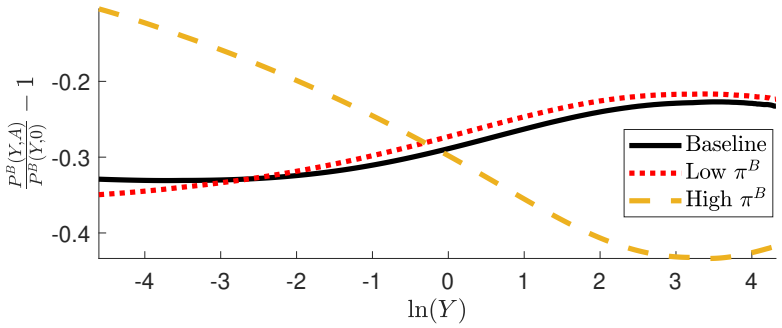

Figure 11: Which countries suffer from digital dollarization? This figure plots the percentage change in currency B's value/adoption when country A successfully launches CBDC in state $z = 0$ (at time T^A). We use our baseline parameters (i.e., $\pi^B = 4$). For the low value of π^B, we pick $\pi^B = 2$, and for the high value of π^B, we pick $\pi^B = 20$.

$P^B(Y, 0)/P_0^B - 1$ which measures the percentage value gain currency B experiences in response to the growth of the cryptocurrency market relative to its initial value P_0^B. This relative value gain is negative for low values of π^B, positive for larger values of π^B, and, notably, highest for high values of π^B. Loosely speaking, the weaker currency B, the more it benefits from the rise of cryptocurrencies.

Consistent with Duffie (2021), our findings suggest that small open economies can mitigate the threat of an invasive digital currency through the early adoption of an effective domestic digital currency. In fact, many developing countries may find it optimal to adopt cryptocurrency as a legal means of payment within their country, especially when they do not have high incentives to issue CBDC. A unilateral adoption of cryptocurrency as a legal means of payment in country B increases the usage of cryptocurrencies and thus could be interpreted in our model as an exogenous, positive shock to the convenience yield parameter Y. Again, developing countries (i.e., characterized by high values of π^B) are more likely to benefit from an increase in Y and so have more incentives to adopt cryptocurrency. These findings rationalize that while countries with stronger currencies, such as the United States and China, try to ban and regulate cryptocurrency, developing countries with very weak currencies and high inflation rates do the opposite and adopt cryptocurrency as a means of payment in addition to its fiat currency.

Finally, we examine whether developing countries and particularly small open economies are more prone to digital dollarization than more developed ones. Figure 11 plots the percentage change in P^B when the strong country A launches CBDC (i.e., when z switches from $z = 0$ to $z = A$).

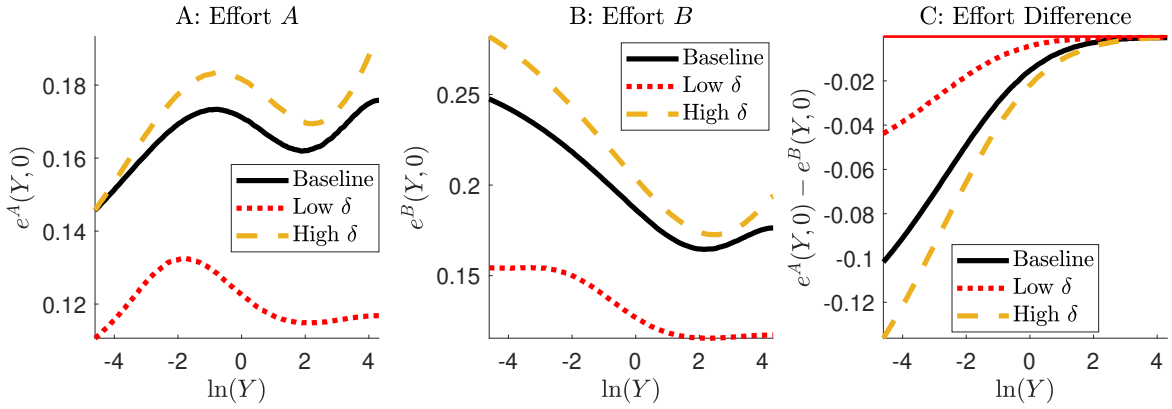

Figure 12: Myopia and Effort. This figure plots countries' optimal efforts $e^x(Y,0)$ as well as their differences agains $\ln Y$ for different values of δ. For the low value of δ, we pick $\delta = 0.1$, and for the high value of π^B, we pick $\delta = 10$.

For low values of Y, the cryptocurrency market is in its infancy, and the degree of dollarization a developing country experiences is massive regardless of whether country A has launched CBDC or not. Under these circumstances, CBDC issuance by the strong country hurts relatively strong non-dominant currencies (low π^B) more than it hurts the weakest currencies of developing countries (high π^B). As discussed previously, the rise of cryptocurrencies benefits developing countries and their currencies the most, while it challenges strong currencies. Once the cryptocurrency market has gained sizeable adoption and $\ln(Y)$ is big, developing countries benefit particularly from reduced competition from A (i.e., less dollarization). Intuitively, the implementation of CBDC by Country A then restores the old currency's dominance with digital dollarization. As such, for larger values of Y, developing countries suffer the most from the implementation of CBDC by the strong country. As Y grows over time, we conclude that in the longer run, developing countries are the most prone to digital dollarization, which is consistent with predictions in Brunnermeier et al. (2019).

5.4 Government Objective, Myopia, and CBDC

We now examine how different government objectives affect countries' incentives to launch CBDC. A parameter of interest in the government objective in (5) is δ which captures countries' focus on the presence or "myopia." We now study how δ affects countries' efforts to digitize their currency.

Figure 12 plots countries' optimal efforts $e^x(Y,0)$ (Panels A and B) as well as their difference (Panel C) against $\ln Y$ in state $z = 0$. As Figure 12 illustrates, a higher value of δ — i.e., higher

focus on the present and myopia — boosts efforts to implement CBDC early on. The reason is that an increase in δ shifts the countries' focus toward the presence, thereby making the first-mover advantage from successfully launching CBDC in state $z = 0$ more appealing. As such, for high levels of δ, countries compete fiercely and exert high effort to launch CBDC in the hope to launch CBDC first. While not explicitly modelled, an increase in myopia δ can therefore be beneficial as it accelerates the digitization of money, thereby simulating potentially valuable financial innovation.

Finally, we emphasize that one could easily generalize the government objective from (5) without loosing tractability, possibly, to take into account other considerations than currency usage, value, or adoption (e.g, geopolitical considerations). For instance, one could set in (5):

$$g_t^x(m_t, P_t) = \beta_0 P_t^x - \beta_{-x} P_t^{-x} - \beta_C P_t^C, \tag{16}$$

so that country x not only cares about adoption/usage/value P_t^x of its own currency, but also explicitly about adoption/usage/value P_t^{-x} of the other country $-x$ or of cryptocurrency. The assumption that country x explicitly cares about the competing currency being used less or being weaker (due to $\beta_{-x} < 0$) could reflect geopolitical considerations.[33] Also observe that the government objective from (5) may capture any economic, fiscal, or geopolitical considerations of country x, in as much these are reflected in the usage, adoption, strength, or value of currencies. However, as we do not explicitly model a country's economy or geopolitical consideration, we leave a careful study of this topic for future research.

5.5 Discussion: Money Supply and Monetary Neutrality

Our assumptions that money holdings do not bear interest and that the supply of currency x is fixed to one are for simplicity and not essential for our key findings. The reason is that our framework can accommodate monetary neutrality. With fixed unit supply, the value (market capitalization) of currency x equals its price in terms consumption, and is denoted by P_t^x. If currency x were not in unit supply, then the price of unit of currency x, denoted by p_t^x, would generally differ from currency value x, denoted by P_t^x.

Notably, our framework features monetary neutrality:[34] If the supply of currency x changes

[33] We expect the results under the stipulation of (16) to be similar to the ones from the baseline, as (2) already provides a tight link between currency values (i.e., large P_t^x means all else equal low P_t^{-x} and P_t^C).

[34] An implicit assumption underlying this result is that the inflation tax rate $\tau_t^x = \kappa^x + \pi^x P_t^A$ depends on the value/strength P_t^A rather than the price level p_t^A of the reserve currency A. We stipulate $\tau_t^x = \kappa^x + \pi^x P_t^x$, as

by a factor ω, the price of currency x in terms of the consumption good changes by a factor $1/\omega$, while the total value of all currency x outstanding remains unchanged at P_t^x. In particular, if the supply of currency x changes by a factor ω and the proceeds from this supply change are distributed pro-rata among the holders of currency x via interest payments (if the proceeds are positive) or taxes (if the proceeds are negative), then the real value of any household's currency x holdings and thus the household's utility remain intact.

In addition, note that with monetary neutrality, it is always possible to transform changes in currency value, dP_t^x, into a tax or interest payment or vice versa.[35] In other words, changes in currency price can be arbitrarily transformed into changes in currency supply and interest payments or taxes for currency holders and vice versa in a way that leaves real quantities and real returns to holding currency x unchanged. As a result, the taxes τ_t^x country x levies on its currency holders can be interpreted as depreciation or inflation of currency x. Under any of these transformations, P_t^x denotes the (total) value of currency x (i.e., the market capitalization of currency x) in terms of the consumption good. In particular, it is possible to peg the price of currency x to one unit of the consumption good. In our setting, even if we allow CBDCs to be interest-bearing, remuneration would not mitigate the currency devaluation against the consumption goods or inflation.

The above logic also extends to cryptocurrencies. An appropriate fee (i.e., tax) and interest payment schedule could implement the price of cryptocurrency being pegged to the price of currency A (e.g., USD) in a way that leaves the real returns to holding cryptocurrency unchanged. In practice, such a peg would pertain for stablecoins (e.g., Tether or DAI).

6 Conclusions

We develop a dynamic model of global currency competition entailing national fiat currencies, cryptocurrencies (stablecoins included), and Central Bank Digital Currencies (CBDCs). The strength of a country's economic fundamentals and the strength of its currency are mutually reinforcing,

we would like to capture that when currency B is weak relative to the consumption good or the dominant/reserve currency, then currency B suffers from high inflation going forward. Our key results go through under alternative specifications for τ_t^x.

[35] For example, when currency x, which is in unit supply, appreciates (i.e., $dP_t^x > 0$) so that at $t + dt$, total value of currency x reads $P_t^x + dP_t^x$, then country x could issue additional dP_t^x/P_t^x units of currency x to drive down currency-x price P_{t+dt}^x to P_t^x, while leaving the total value of currency x at time $t + dt$, i.e., $P_t^x(1 + dP_t^x/P_t^x) = P_t^x + dP_t^x$, unchanged. The proceeds from this supply change is dP_t^x units of the consumption good. The country pays these proceeds to currency holders as interest payments on currency x on a pro-rata basis to its currency holders, yielding interest payments of dP_t^x units of the consumption good per unit of currency x.

leading to global currency dominance by the strongest countries. The endogenous rise of cryptocurrencies hurts the stronger currency, but may benefit weaker currencies by reducing fiat competition and dollarization. Reserve requirements on stablecoins mitigate the impact of cryptocurrencies on the fiat currencies they are pegged to. Our findings suggest that the U.S. and the U.S. dollar can potentially benefit from regulation that requires U.S. dollar stablecoins to be backed by U.S. dollar reserves, so as to seize part of the seigniorage from stablecoin issuance.

Because countries' strategic decisions to implement CBDCs reflect the competition from both emergent cryptocurrencies and other fiat currencies, a pecking order for digital currency development emerges: Countries with strong but non-dominant currencies (e.g., China and Switzerland) tend to have the highest-powered incentives to launch CBDC first so as to attain a technological and cumulative first-mover advantage; countries with dominant currencies (e.g., the United States and European Union) are motivated to launch CBDC early on both to nip cryptocurrency growth in the bud and later to counteract a competitor's CBDC; nations with the weakest or without a sovereign currency may opt for cryptocurrencies or stablecoins pegged to a basket of currencies or a consumption index to avoid (digital) dollarization. In general, weaker national currencies imply a vacuum in the currency space and so favor the emergence of cryptocurrencies as competitors and boost countries' incentives to implement CBDC, both spurring valuable financial innovations.

As a first study on currency competition and strategic digitization, we necessarily abstracted away from several important dimensions concerning cryptocurrencies and CBDCs such as the design of CBDCs and how cryptocurrencies derive value. We also had to stipulate in our model the governments' objectives and limitations in response actions to the crypto sector, which should be explored in future research. Nevertheless, our findings help rationalize recent developments in the digitization of money and payment innovations, while offering insights into the future of money and the global battle of both digital and conventional currencies.

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Appendix

A Solution to the Dynamic Model and Proof of Proposition 1

We solve for the equilibrium in several parts. Part I characterizes the market clearing conditions. Part II discusses the household optimization. Part III solves for the currency values and adoption as a function of (Y, z). Part IV solves for the government value function as a function (Y, z). Part V summarizes the systems of coupled ODEs that characterize the Markov equilibrium. Throughout, we assume that a unique Markov equilibrium with state variables (Y, z) exists. A formal uniqueness and existence proof is beyond the scope of the paper.

A.1 Part I — Market Clearing Conditions

To begin with, note the market clearing conditions, $m_t^B = P_t^B$ and $m_t^C = P_t^C$, for currencies B and C respectively. Recall that fraction θ of cryptocurrency value P_t^C is backed by currency A reserves, where $\theta \in [0, 1)$ is an exogenous constant. This way, our model can accommodate dollar-backed stablecoins, such as USDC, because we associate currency A with the U.S. dollar.

As a result, total reserves backing cryptocurrency are worth θP_t^C units of the consumption good. Thus, the reserves backing cryptocurrency consist of $\theta P_t^C / P_t^A$ units of currency A, leaving the circulating supply of currency A at $(1 - \theta P_t^C / P_t^A)$ units. For the market for currency A to clear, the household holds this circulating supply, i.e.,

$$m_t^A / P_t^A = 1 - \theta P_t^C / P_t^A$$

units of currency A. Therefore, the household's holdings of currency A in units of the consumption good is:

$$m_t^A = P_t^A - \theta P_t^C. \tag{17}$$

The condition (1), i.e., $m_t^A + m_t^B + m_t^C = 1$, then becomes:

$$P_t^A + P_t^B + P_t^C(1 - \theta) = 1. \tag{18}$$

Thus, we can solve for:

$$P_t^C = \frac{1 - P_t^A - P_t^B}{1 - \theta} \tag{19}$$

and, inserting P_t^C into (17), we obtain

$$m_t^A = P_t^A - \theta P_t^C = P_t^A - \frac{\theta(1 - P_t^A - P_t^B)}{1 - \theta} = \frac{P_t^A - \theta(1 - P_t^B)}{1 - \theta}, \tag{20}$$

which is the market clearing condition for currency A.

A.2 Part II — Household Optimization

To begin with, we introduce the "CBDC state variable:" $z_t = z = 0$ denotes that no country has launched up to time t; $z_t = z = A$ ($z_t = z = A$) denotes that only country A (B) has launched CBDC by time t; and, $z_t = z = AB$ means that both countries have launched CBDC by time t. we postulate that equilibrium currency prices $P_t^x = P^x(Y, z)$ for $(Y, z) = (Y_t, z_t)$ follow the law of motion:

$$\frac{dP_t^x}{P_t^x} = \mu^x(Y, z)dt + \Delta^x(Y, z; z')dJ_t^{z,z'}, \tag{21}$$

where $\mu^x(Y, z)$ is the endogenous price drift in state $(Y_t, z_t) = (Y, z)$. In (21), $\Delta^x(Y, z; z')$ is the endogenous (percentage) value change of currency x if the CBDC state changes from z to z'. The jump process $dJ_t^{z,z'} \in \{0, 1\}$ equals one if and only if the CBDC state changes from z to z' at time t; otherwise, $dJ_t^{z,z'} = 0$.

Recall the definition of expected currency returns in terms of the consumption good:

$$r_t^x := \frac{\mathbb{E}[dP_t^x]}{P_t^x dt}.$$

We can then write cohort t's consumption c_{t+dt} at $t + dt$ as

$$c_{t+dt} = \sum_{x\in\{A,B,C\}} \frac{m_t^x P_{t+dt}^x}{P_t^x} - \sum_{x\in\{A,B\}} \frac{\tau_t^x m_t^x}{P_t^x} dt, \tag{22}$$

whereby — as discussed in the main text — "taxes" τ_t^x take the form $\tau_t^x = \kappa^x + \pi^x P_t^A$.

Observe that $P_{t+dt}^x = P_t^x + dP_t^x$. Because the representative household uses its entire endowment one to buy currencies at time t, $\sum_{x\in\{A,B,C\}} m_t^x = 1$. We can thus rewrite (22) as:

$$c_{t+dt} = 1 + \sum_{x\in\{A,B,C\}} \frac{m_t^x dP_t^x}{P_t^x} - \sum_{x\in\{A,B\}} \frac{\tau_t^x m_t^x}{P_t^x} dt. \tag{23}$$

Now, note that the representative household maximizes her lifetime utility

$$\max_{m_t^x \geq 0} \mathbb{E}[U_t] \quad \text{s.t.} \quad \sum_{x\in\{A,B,C\}} m_t^x = 1, \tag{24}$$

taking prices P_t^x as given. Here, the lifetime payoff U_t reads:

$$U_t = c_{t+dt} + Z_o(m_t^A + m_t^B + m_t^C)dt + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt,$$

so that

$$\begin{aligned}\mathbb{E}[U_t] =& 1 - \sum_{x\in\{A,B\}} \frac{\tau_t^x m_t^x}{P_t^x} dt + \sum_{x\in\{A,B,C\}} m_t^x r_t^x dt \\ &+ Z_o(m_t^A + m_t^B + m_t^C)dt + Z_t^A v(m_t^A)dt + Z_t^B v(m_t^B)dt + Y_t v(m_t^C)dt.\end{aligned} \tag{25}$$

In light of $\sum_{x\in\{A,B,C\}} m_t^x = 1$ and (25), the solution (m_t^A, m_t^B, m_t^C) to (24) satisfies

$$(m_t^A, m_t^B, m_t^C) = \arg\max_{m_t^x \geq 0} \Omega(m_t^A, m_t^B, m_t^C) \quad \text{s.t.} \quad \sum_{x\in\{A,B,C\}} m_t^x = 1,$$

with

$$\Omega(m_t^A, m_t^B, m_t^C) := \sum_{x\in\{A,B,C\}} m_t^x r_t^x - \sum_{x\in\{A,B\}} \frac{\tau_t^x m_t^x}{P_t^x} + Z_t^A v(m_t^A) + Z_t^B v(m_t^B) + Y_t v(m_t^C).$$

Notice that

$$\Omega(m_t^A, m_t^B, m_t^C) = constant + \frac{\mathbb{E}[U_t]}{dt}, \tag{26}$$

where *constant* does not depend on m_t^x. In light of $\sum_{x\in\{A,B,C\}} m_t^x = 1$, it must hold in optimum that the household is indifferent between substituting a marginal unit of any currency for another one, i.e.,

$$\frac{\partial\Omega(m_t^A, m_t^B, m_t^C)}{\partial m_t^A} = \frac{\partial\Omega(m_t^A, m_t^B, m_t^C)}{\partial m_t^B} = \frac{\partial\Omega(m_t^A, m_t^B, m_t^C)}{\partial m_t^C}, \tag{27}$$

provided $m_t^x \in (0,1)$. Note that because of (26), condition (27) becomes equivalent to (10) from the main text, as desired.

Taking the derivative in (27) and using the definition of $\Omega(m_t^A, m_t^B, m_t^C)$, we calculate

$$\begin{aligned} Y_t v'(m_t^C) + r_t^C &= Z_t^A v'\left(m_t^A\right) + r_t^A - \frac{\tau_t^A}{P_t^A} \\ Y_t v'(m_t^C) + r_t^C &= Z_t^B v'(m_t^B) + r_t^B - \frac{\tau_t^B}{P_t^B}, \end{aligned} \tag{28}$$

Inserting the market clearing condition $m_t^A = \frac{P_t^A - \theta(1-P_t^B)}{1-\theta}$ from (20), $m_t^B = P_t^B$, and $m_t^C = P_t^C$ into (28), we obtain

$$\begin{aligned} Y_t v'(P_t^C) + r_t^C &= Z_t^A v'\left(\frac{P_t^A - \theta(1-P_t^B)}{1-\theta}\right) + r_t^A - \frac{\tau_t^A}{P_t^A} \\ Y_t v'(P_t^C) + r_t^C &= Z_t^B v'(P_t^B) + r_t^B - \frac{\tau_t^B}{P_t^B}. \end{aligned} \tag{29}$$

Notice that (28) is equivalent to (11). Because $\lim_{m_t^x \to 0} m_t^x v'(m_t^x) = \infty$, any solution to (11) or (29) must satisfy $P_t^x \in (0,1)$. Also note that the constant base (marginal) convenience Z_o, which

is the same across all currencies, does not enter the equilibrium pricing condition (11) or (29). As such, the exact value of Z_o does not affect equilibrium quantities P_t^x, m_t^x, or V_t^x.

A.3 Part III — Solving for Currency Values

We now express the currency values P_t^x and currency returns r_t^x as well as the countries' efforts to implement CBDC e_t^x as functions of Y and state $z \in \{0, A, B, AB\}$, and we omit time subscripts unless necessary.

We conjecture and verify that $P_t^x = P(Y_t, z_t)$, $m_t^x = m^x(Y_t, z_t)$, and $e_t^x = e(Y_t, z_t)$ for $x = A, B, C$, with functions $P^x(\cdot)$, $m^x(\cdot)$, and $e^x(\cdot)$. It then follows that r_t^x is a function of (Y, z) too, in that $r_t^x = r^x(Y, z)$. Also write $dY = \mu^Y(Y, z)dt$ whereby the drift of dY reads according to (4):

$$\mu^Y(Y,z) = \begin{cases} \mu Y m^C(Y,z) & \text{if} \quad Y < \overline{Y} \\ 0 & \text{if} \quad Y = \overline{Y}. \end{cases} \tag{30}$$

Next, market clearing in equilibrium implies $P_t^x = P^x(Y, z) = m_t^x = m^x(Y, z)$ for $x \in \{B, C\}$, and, according to (20):

$$m_t^A = m^A(Y,z) = \frac{P^A(Y,z) - \theta(1 - P^B(Y,z))}{1-\theta}.$$

Also, we get from (18):

$$P^A(Y,z) + P^B(Y,z) + P^C(Y,z)(1-\theta) = 1 \quad \Longleftrightarrow \quad P^C(Y,z) = \frac{1 - P^A(Y,z) - P^B(Y,z)}{1-\theta}.$$

Recall that according to (21):

$$\frac{dP_t^x}{P_t^x} = \frac{dP^x(Y,z)}{P^x(Y,z)} = \mu^x(Y,z)dt + \Delta^x(Y,z;z')dJ^{z,z'},$$

where $\mu^x(Y, z)$ is the endogenous price drift in state $(Y_t, z_t) = (Y, z)$. $\Delta^x(Y, z; z')$ is the endogenous (percentage) value change of currency x if the CBDC state changes from z to z'. The jump process $dJ_t^{z,z'} \in \{0, 1\}$ equals one if and only if the CBDC state changes from z to z' at time t; otherwise, $dJ_t^{z,z'} = 0$. Notice that:

$$\Delta^x(Y,z;z') = \frac{P^x(Y,z')}{P^x(Y,z)} - 1,$$

and $\Delta^x(Y, z; z')P^x(Y, z) = P^x(Y, z') - P^x(Y, z)$.

Denote

$$(P^x)'(Y,z) = \frac{\partial}{\partial Y} P^x(Y,z).$$

For $Y = \overline{Y}$, the price drifts $\mu_t^x = \mu^x(Y, z)$ from (21) equal zero, as the drift of dY equals zero once

Y reaches $\overline{Y}$. By Ito's Lemma, the drift of currency value x, that is, $\mu^x(Y,z)$, becomes

$$\mu^x(Y,z) = \left(\frac{(P^x)'(Y,z)}{P^x(Y,z)}\right)\mu^Y(Y,z), \tag{31}$$

where $\mu^Y(Y,z)$ is the drift of dY from (30) (which vanishes for $Y=\overline{Y}$).

Also note that because $P_t^A+P_t^B+P_t^C(1-\theta)=1$ (i.e., $P^Y(Y,z)+P^B(Y,z)+P^C(Y,z)(1-\theta)=1$), we have $dP_t^A+dP_t^B+dP_t^C(1-\theta)=0$, which implies by means of (21)

$$\mu^A(Y,z)P^A(Y,z)+\mu^B(Y,z)P^B(Y,z)+\mu^C(Y,z)P^C(Y,z)(1-\theta)=0 \tag{32}$$

as well as

$$\Delta^A(Y,z,z')P^A(Y,z)+\Delta^B(Y,z,z')P^B(Y,z)+\Delta^C(Y,z,z')P^C(Y,z)(1-\theta)=0. \tag{33}$$

In light of (32), (33), or $P_t^A+P_t^B+P_t^C(1-\theta)=1$, it suffices to characterize the currency values and dynamics for currencies A and B, and the value and the dynamics for currency C follow as the residual, and can be backed out knowing $P^A(Y,z)$ and $P^B(Y,z)$ (and their dynamics).

Next, we can characterize expected returns r_t^x, and write $r_t^x=r^x(Y,z)$. It holds that:

$$\begin{aligned}
r^x(Y,0) &= \mu^x(Y,0)+\lambda e^A(Y,0)\big(P^x(Y,A)/P^x(Y,0)-1\big)+\lambda e^B(Y,0)\big(P^x(Y,B)/P^x(Y,0)-1\big),\\
r^x(Y,A) &= \mu^x(Y,A)+\lambda e^B(Y,A)\big(P^x(Y,AB)/P^x(Y,A)-1\big),\\
r^x(Y,B) &= \mu^x(Y,B)+\lambda e^A(Y,B)\big(P^x(Y,AB)/P^x(Y,B)-1\big),\\
r^x(Y,AB) &= \mu^x(Y,AB).
\end{aligned} \tag{34}$$

We already used that $e^x(Y,x)=e^x(Y,AB)=0$ for $x=A,B$, i.e., there is no more effort by country x after it has successfully launched CBDC (at time T^x). Combining (32), (33), and (34), we also obtain

$$r^A(Y,z)P^A(Y,z)+r^B(Y,z)P^B(Y,z)+r^C(Y,z)P^C(Y,z)(1-\theta)=0. \tag{35}$$

The equilibrium condition (29) yields for $x=A,B$:

$$Yv'\big(P^C(Y,z)\big)+r^C(Y,z)=Z^x(Y,z)v'\big(m^x(Y,z)\big)+r^x(Y,z)-\frac{\tau^x(Y,z)}{P^x(Y,z)}, \tag{36}$$

where $Z^A(Y,z)=Z_L$ for $z=0,B$ and $Z^A(Y,z)=Z_H+\alpha Y$ for $z=A,AB$. Likewise, $Z^B(Y,z)=Z_L$ for $z=0,A$ and $Z^B(Y,z)=Z_H+\alpha Y$ for $z=B,AB$. Note that by (20), $m^A(Y,z)=\frac{P^A(Y,z)-\theta(1-P^B(Y,z))}{1-\theta}$, and $m^B(Y,z)=P^B(Y,z)$. We also know that $\tau^A(Y,z)=\kappa^A+\pi^AP^A(Y,z)$ and $\tau^B(Y,z)=\kappa^B+\pi^BP^A(Y,z)$.

As a result, under the assumption that optimal effort e_t^x is a function of (Y,z) (i.e., $e_t^x=e^x(Y,z)$), we have verified that all model equilibrium quantities can be expressed in terms of (Y,z). The next Part IV shows that indeed, optimal effort e_t^x is a function of (Y,z).

A.4 Part IV: Solving Government Objective

At a given time t, the government x chooses effort $(e_s^x)_{s\geq t}$ to maxmize the objective function V_t^x as follows:

$$V_t^x = \max_{(e_s^x)_{s\geq t}} \mathbb{E}_t^x \left[\int_t^\infty e^{-\delta(s-t)} \left(\beta\delta P_s^x - \frac{(e_s^x)^2}{2} \right) ds \right], \tag{37}$$

for constants $\beta, \delta \geq 0$.

By the dynamic programming principle, the government's value function solves the following HJB equation:

$$\delta V_t^x = \max_{e_t^x \geq 0} \left(\beta\delta P_t^x - \frac{(e_t^x)^2}{2} + \frac{\mathbb{E}_t^x[dV_t^x]}{dt} \right), \tag{38}$$

which is (12). Notice that the expectation $\mathbb{E}_t^x[dV_t^x]$ depends on the levels of (e^A, e^B) and is conditional on country x's time$-t$ information (which includes time-t public information and e^x); country x takes the effort of the other country $-x$ as given. Effort e_t^x is not observable for the household or the competing country, and countries cannot commit to effort levels. As such, the choice of effort e_t^x at any time t is privately optimal. Clearly, effort e_t^x is redundant after time T^x, i.e., after country x has implemented CBDC. As such, we set $e^x(Y, x) = e^x(Y, AB) = 0$ for $x = A, B$.

Next, we can express V_t^x as a function of (Y, z) only, i.e., $V_t^x = V^x(Y_t, z_t)$. Further, we solve for efforts $e_t^x = e^x(Y, z)$ and derive eight first order ODEs that characterize the functions $V^x(Y, AB)$. To do so, we now consider all states $z = 0, A, B, AB$ separately. In what follows, x is either A or B. When $x = A$, then $-x = B$ and vice versa (i.e., when $x = B$, then $-x = A$). In what follows, we suppress the dependence of $\mathbb{E}_t^x$ on (x, t) and simply write $\mathbb{E}$ for the expectation. Likewise, we suppress time subscripts, unless confusion arises.

To simplify notation, we define

$$(V^x)'(Y, z) := \frac{\partial V^x(Y, z)}{\partial Y}.$$

A.4.1 State $z = AB$

Clearly, $e^x(Y, AB) = 0$. Using Ito's Lemma, we can calculate

$$\frac{\mathbb{E}[dV^x(Y, AB)]}{dt} = (V^x)'(Y, AB)\mu^Y(Y, AB),$$

where $\mu^Y(Y, z)$ is the drift of dY from (30). Inserting these relations into (38), we obtain

$$\delta V^x(Y, AB) = \beta\delta P^x(Y, AB) + (V^x)'(Y, AB)\mu^Y(Y, AB), \tag{39}$$

which are two first order ODEs in Y, given $z = AB$.

A.4.2 State $z = x$

Consider state $z = x$ for $x = A$ or $x = B$. Recall that when $x = A$, then $-x = B$ and vice versa. Then, $e^x(Y, x) = 0$. Using Ito's Lemma for jump processes, we can calculate

$$\frac{\mathbb{E}[dV^x(Y,z)]}{dt} = (V^x)'(Y,x)\mu^Y(Y,x) + \lambda e^{-x}(Y,x)(V^x(Y,AB) - V^x(Y,x)), \tag{40}$$

and

$$\frac{\mathbb{E}[dV^{-x}(Y,z)]}{dt} = (V^{-x})'(Y,x)\mu^Y(Y,z) + \lambda e^{-x}(Y,x)(V^{-x}(Y,AB) - V^{-x}(Y,x)), \tag{41}$$

Inserting (41) into (38) for country $-x$, we obtain

$$\begin{aligned}\delta V^{-x}(Y,x) = \max_{e^{-x}(Y,x)\geq 0}\Big\{&\beta\delta P^{-x}(Y,x) + (V^{-x})'(Y,x)\mu^Y(Y,x) \\ &+ \lambda e^{-x}(Y,z)(V^{-x}(Y,AB) - V^{-x}(Y,x)) - \frac{(e^{-x}(Y,x))^2}{2}\Big\}.\end{aligned} \tag{42}$$

The optimization with respect to effort $e^{-x}(Y,x)$ yields (with some abuse of notation)

$$e^{-x}(Y,x) = \lambda(V^{-x}(Y,AB) - V^{-x}(Y,x)). \tag{43}$$

Reinserting optimal effort into (44) yields

$$\delta V^{-x}(Y,x) = \beta\delta P^{-x}(Y,x) + (V^{-x})'(Y,x)\mu^Y(Y,x) + \frac{\lambda^2 e^{-x}(Y,x)(V^{-x}(Y,AB) - V^{-x}(Y,x))^2}{2}. \tag{44}$$

Performing similar steps for country x, we have

$$\delta V^x(Y,x) = \delta\beta P^x(Y,x) + (V^x)'(Y,x)\mu^Y(Y,x) + \lambda e^{-x}(Y,x)(V^x(Y,AB) - V^x(Y,x)).$$

Above is equivalent to

$$\begin{aligned}\delta V^x(Y,z) = &\beta\delta P^x(Y,x) + (V^{-x})'(Y,x)\mu^Y(Y,x) \\ &+ \lambda^2(V^{-x}(Y,AB) - V^{-x}(Y,x))(V^x(Y,AB) - V^x(Y,x)).\end{aligned} \tag{45}$$

A.4.3 State $z = -x$

The analysis of state $z = -x$ is analogous to the analysis of state x and follows from replacing x by $-x$.

A.4.4 State $z = 0$

In state $z = 0$, we can calculate for $x = A, B$:

$$\begin{aligned}\mathbb{E}[dV^x(Y,z)] = (V^x)'(Y,0)\mu^Y(Y,0) \\ + \lambda e^x(Y,0)(V^x(Y,x) - V^x(Y,0)) + \lambda e^{-x}(Y,0)(V^x(Y,-x) - V^x(Y,0)).\end{aligned} \tag{46}$$

We can now insert (46) into (38) and obtain (after omitting time subscripts) in state $(Y,0)$ for $x = A, B$:

$$\begin{aligned}\delta V^x(Y,0) = \max_{e^x(Y,0)\geq 0} \Big\{ \beta\delta P^x(Y,0) - \frac{(e^x(Y,0))^2}{2} + (V^x)'(Y,0)\mu^Y(Y,0) \\ + \lambda e^x(Y,0)(V^x(Y,x) - V^x(Y,0)) + \lambda e^{-x}(Y,0)(V^x(Y,-x) - V^x(Y,0)) \Big\}.\end{aligned}$$

The optimization with respect to effort in state $z = 0$ as

$$e^x(Y,0) = \lambda(V^x(Y,x) - V^x(Y,0)). \tag{47}$$

Reinserting optimal effort, we obtain for $z = 0$ and $x = A, B$:

$$\begin{aligned}\delta V^x(Y,0) =& \beta\delta P^x(Y,0) + (V^x)'(Y,0)\mu^Y(Y,0) \\ &+ \frac{\lambda^2(V^x(Y,x) - V^x(Y,0))^2}{2} + \lambda^2(V^{-x}(Y,-x) - V^{-x}(Y,0))(V^x(Y,-x) - V^x(Y,0)).\end{aligned} \tag{48}$$

A.5 Part V: System of ODEs

To get a better overview, we now explicitly gather the ODEs that characterize the Markov equilibrium by collecting and summarizing our findings from Parts I through IV. We separately consider the states $z = 0$, $z = x \in \{A, B\}$, and $z = AB$, starting with state $z = AB$.

Next, recall that

$$\begin{aligned}m^A(Y,z) &= \frac{P^A(Y,z) - \theta(1 - P^B(Y,z))}{1-\theta} \\ m^B(Y,z) &= P^B(Y,z) \\ m^C(Y,z) &= P^C(Y,z) = \frac{1 - P^A(Y,z) - P^B(Y,z)}{1-\theta}.\end{aligned} \tag{49}$$

These relations will be used throughout for any $z \in \{0, A, B, AB\}$.

A.5.1 State $z = AB$

In state $z = AB$, we combine (34), (31), and (35) to calculate

$$\begin{aligned}
r^A(Y, AB) &= \left(\frac{(P^A)'(Y, AB)}{P^A(Y, AB)} \right) \mu^Y(Y, AB) \\
r^B(Y, AB) &= \left(\frac{(P^B)'(Y, AB)}{P^B(Y, AB)} \right) \mu^Y(Y, AB) \\
r^C(Y, AB) &= -\left(\frac{r^A(Y, AB)P^A(Y, AB) + r^B(Y, AB)P^B(Y, AB)}{1 - P^B(Y, AB) - P^B(Y, AB)} \right)
\end{aligned}$$

Then, (36) implies

$$\begin{aligned}
Yv'\left(m^C(Y, AB)\right) + r^C(Y, AB) &= (Z_H + \alpha Y)v'\left(m^A(Y, AB)\right) + r^A(Y, AB) - \frac{\tau^A(Y, AB)}{P^A(Y, AB)} \\
Yv'\left(m^C(Y, AB)\right) + r^C(Y, AB) &= (Z_H + \alpha Y)v'\left(m^B(Y, AB)\right) + r^B(Y, AB) - \frac{\tau^B(Y, AB)}{P^B(Y, AB)}.
\end{aligned} \tag{50}$$

And, from (39), we know

$$\begin{aligned}
\delta V^A(Y, AB) =& \beta\delta P^A(Y, AB) + (V^A)'(Y, AB)\mu^Y(Y, AB), \\
\delta V^B(Y, AB) =& \beta\delta P^B(Y, AB) + (V^B)'(Y, AB)\mu^Y(Y, AB).
\end{aligned} \tag{51}$$

At the boundary $Y = \overline{Y}$, the drift of dY vanishes (i.e., $\mu^Y(\overline{Y}, z) = 0$), and the solution, that is, $(P^x, (\overline{Y}, AB), V^x, (\overline{Y}, AB))$ for $x = AB$, is characterized by the following system of four equations

$$\begin{aligned}
\overline{Y}v'\left(m^C(\overline{Y}, AB)\right) &= (Z_H + \alpha\overline{Y})v'\left(m^A(\overline{Y}, AB)\right) - \frac{\tau^A(\overline{Y}, AB)}{P^A(\overline{Y}, AB)} \\
\overline{Y}v'\left(m^C(\overline{Y}, AB)\right) &= (Z_H + \alpha\overline{Y})v'\left(m^B(\overline{Y}, AB)\right) - \frac{\tau^B(\overline{Y}, AB)}{P^B(\overline{Y}, AB)} \\
\delta V^A(\overline{Y}, AB) &= \beta\delta P^A(\overline{Y}, AB) \\
\delta V^B(\overline{Y}, AB) &= \beta\delta P^B(\overline{Y}, AB).
\end{aligned} \tag{52}$$

Clearly, one can solve (52) for the four unknowns $P^A(\overline{Y}, AB)$, $P^B(\overline{Y}, AB)$, $V^A(\overline{Y}, AB)$, and $V^B(\overline{Y}, AB)$. To solve for the Markov equilibrium in state $z = AB$, we first solve the system of equations in (52) to obtain $(P^x(\overline{Y}, AB), V^x(\overline{Y}, AB))$ for $x = A, B$.

Then, we solve the system of four coupled first order ODEs in (50) and (51) subject to the boundary conditions/boundary values $(P^x, (\overline{Y}, AB), V^x, (\overline{Y}, AB))$, which then yields prices $P^x(Y, AB)$ for $x = A, B$ as well as $P^C(Y, AB)$ via $P^C(Y, AB) = m^C(Y, AB) = \frac{1-P^A(Y,z)-P^B(Y,z)}{1-\theta}$.

Finally, note that the Picard-Lindeloef theorem implies that, under mild regularity conditions, the system of first order ODEs (50) and (51) admits a unique solution, given the boundary values $(P^x(\overline{Y}, AB), V^x(\overline{Y}, AB))$. As such, the Markov equilibrium in state $z = AB$ is unique as long as

(52) admits a unique solution $(P^x(\overline{Y}, AB), V^x(\overline{Y}, AB))$ for $x = A, B$ (and the conditions of the Picard-Lindeloef theorem are satisfied).

A.5.2 State $z = x \in \{A, B\}$

In state $z = x$, we have $e^x(Y, x) = 0$ and $e^{-x}(Y, x) = \lambda(V^{-x}(Y, AB) - V^{-x}(Y, x))$. Then, we can combine (34), (31), and (35) to obtain

$$
\begin{aligned}
r^A(Y,x) &= \left(\frac{(P^A)'(Y,x)}{P^A(Y,x)}\right)\mu^Y(Y,x) + \lambda e^{-x}(Y,x)\left(\frac{P^A(Y,AB)}{P^A(Y,x)} - 1\right).\\
r^B(Y,x) &= \left(\frac{(P^B)'(Y,x)}{P^B(Y,x)}\right)\mu^Y(Y,x) + \lambda e^{-x}(Y,x)\left(\frac{P^B(Y,AB)}{P^B(Y,x)} - 1\right)\\
r^C(Y,x) &= -\left(\frac{r^A(Y,x)P^A(Y,x) + r^B(Y,x)P^B(Y,x)}{1 - P^B(Y,x) - P^B(Y,x)}\right)
\end{aligned}
$$

Then, (36) implies

$$
\begin{aligned}
Yv'\left(m^C(Y,x)\right) + r^C(Y,x) &= Z^A(Y,z)v'\left(m^A(Y,x)\right) + r^A(Y,x) - \frac{\tau^A(Y,x)}{P^A(Y,x)}\\
Yv'\left(m^C(Y,x)\right) + r^C(Y,x) &= Z^B(Y,x)v'\left(m^B(Y,x)\right) + r^B(Y,x) - \frac{\tau^B(Y,x)}{P^B(Y,x)},
\end{aligned}
\tag{53}
$$

where $Z^A(Y,z) = Z_L$ for $z = 0, B$ and $Z^A(Y,z) = Z_H + \alpha Y$ for $z = A, AB$. Likewise, $Z^B(Y,z) = Z_L$ for $z = 0, A$ and $Z^B(Y,z) = Z_H + \alpha Y$ for $z = B, AB$. And, from (45) and (44), we know

$$
\begin{aligned}
\delta V^{-x}(Y,x) =& \beta\delta P^{-x}(Y,z) + (V^{-x})'(Y,x)\mu^Y(Y,x) + \frac{\lambda^2(V^{-x}(Y,AB) - V^{-x}(Y,x))^2}{2}\\
\delta V^{x}(Y,z) =& \beta\delta P^{x}(Y,z) + (V^{x})'(Y,x)\mu^Y(Y,x)\\
&+ \lambda^2(V^{-x}(Y,AB) - V^{-x}(Y,x))(V^x(Y,AB) - V^x(Y,x)).
\end{aligned}
\tag{54}
$$

To solve the model for the Markov equilibrium in state $z = x$, we need to solve the system of four coupled first order ODEs, which is characterized in (53) and (54), for $P^A(Y,x)$, $P^B(Y,x)$, $V^A(Y,x)$, and $V^B(Y,x)$. Given the solution, we then also obtain $P^C(Y,x) = m^C(Y,x) = \frac{1-P^A(Y,x)-P^B(Y,x)}{1-\theta}$.

At the boundary $Y = \overline{Y}$, the drift of dY vanishes so that the system characterized in (53) and (54) becomes a system of four non-linear equations, which can be solved for the four unknowns $P^A(\overline{Y},x)$, $P^B(\overline{Y},x)$, $V^A(\overline{Y},x)$, and $V^B(\overline{Y},x)$, given the values of $P^A(\overline{Y},AB)$, $P^B(\overline{Y},AB)$, $V^A(\overline{Y},AB)$, and $V^B(\overline{Y},AB)$.

Finally, note that the Picard-Lindeloef theorem implies that, under mild regularity conditions, the system of first order ODEs (53) and (54) admits a unique solution, given the boundary values $(P^{x'}(\overline{Y},x), V^{x'}(\overline{Y},x))$ for $x' = A, B$. As such, the Markov equilibrium in state $z = x$ is unique as long as the boundary values $(P^{x'}(\overline{Y},x), V^{x'}(\overline{Y},x))$ exist and are unique (and the conditions of the Picard-Lindeloef theorem are satisfied). This is the case if (52) admits a

unique solution $(P^x(\overline{Y}, AB), V^x(\overline{Y}, AB))$ for $x = A, B$ and (53) and (54) admit a unique solution $(P^{x'}(\overline{Y}, x), V^{x'}(\overline{Y}, x))$ for $x' = A, B$ at $Y = \overline{Y}$.

A.5.3 State $z = 0$

In state $z = 0$, we have

$$e^A(Y,0) = \lambda(V^A(Y,A) - V^A(Y,0)) \quad \text{and} \quad e^B(Y,0) = \lambda(V^B(Y,B) - V^B(Y,0)).$$

Then, we can combine (34), (31), and (35) to obtain

$$\begin{aligned}
r^A(Y,0) &= \left(\frac{(P^A)'(Y,0)}{P^A(Y,0)}\right)\mu^Y(Y,0) + \lambda \sum_{x=A,B} e^x(Y,0)\left(\frac{P^A(Y,x)}{P^A(Y,0)} - 1\right). \\
r^B(Y,0) &= \left(\frac{(P^B)'(Y,0)}{P^B(Y,0)}\right)\mu^Y(Y,0) + \lambda \sum_{x=A,B} e^x(Y,0)\left(\frac{P^B(Y,x)}{P^B(Y,0)} - 1\right) \\
r^C(Y,0) &= -\left(\frac{r^A(Y,0)P^A(Y,0) + r^B(Y,0)P^B(Y,0)}{1 - P^B(Y,0) - P^B(Y,0)}\right)
\end{aligned}$$

Then, (36) implies

$$\begin{aligned}
Yv'\left(m^C(Y,0)\right) + r^C(Y,0) &= Z_L v'\left(m^A(Y,0)\right) + r^A(Y,0) - \frac{\tau^A(Y,0)}{P^A(Y,0)} \\
Yv'\left(m^C(Y,0)\right) + r^C(Y,0) &= Z_L v'\left(m^B(Y,0)\right) + r^B(Y,0) - \frac{\tau^B(Y,0)}{P^B(Y,)}.
\end{aligned} \tag{55}$$

And, from (48), we know for $x = A, B$:

$$\begin{aligned}
\delta V^x(Y,0) =& \beta\delta P^x(Y,0) + (V^x)'(Y,0)\mu^Y(Y,0) \\
&+ \frac{\lambda^2(V^x(Y,x) - V^x(Y,0))^2}{2} + \lambda^2(V^{-x}(Y,-x) - V^{-x}(Y,0)(V^x(Y,-x) - V^x(Y,0)).
\end{aligned} \tag{56}$$

To solve the model for the Markov equilibrium in state $z = x$, we need to solve the system of four coupled first order ODEs, which is characterized in (55) and (56), for $P^A(Y,0)$, $P^B(Y,0)$, $V^A(Y,0)$, and $V^B(Y,0)$. We then also obtain $P^C(Y,0) = m^C(Y,0) = \frac{1-P^A(Y,0)-P^B(Y,0)}{1-\theta}$.

At the boundary $Y = \overline{Y}$, the drift of dY vanishes so that the system characterized in (55) and (56) becomes a system of non-linear equations, which can be solved for the four unknowns $P^A(\overline{Y},0)$, $P^B(\overline{Y},0)$, $V^A(\overline{Y},0)$, and $V^B(\overline{Y},0)$, given the values of $P^A(\overline{Y},x)$, $P^B(\overline{Y},x)$, $V^A(\overline{Y},x)$, and $V^B(\overline{Y},x)$ for $x = A, B$.

Finally, note that the Picard-Lindeloef theorem implies that, under mild regularity conditions, the system of first order ODEs (55) and (56) admits a unique solution, given the boundary values $(P^{x'}(\overline{Y},0), V^{x'}(\overline{Y},0))$ for $x' = A, B$. As such, the Markov equilibrium in state $z = 0$ is unique as long as the boundary values are $(P^{x'}(\overline{Y},0), V^{x'}(\overline{Y},0))$ exist and are unique (and the

conditions of the Picard-Lindeloef theorem are satisfied). This is the case if (i) (52) admits a unique solution $(P^x(\overline{Y}, AB), V^x(\overline{Y}, AB))$ for $x = A, B$, (ii) (53) and (51) admit a unique solution $(P^{x'}(\overline{Y}, x), V^{x'}(\overline{Y}, x))$ for $x' = A, B$ and $x = A, B$ at $Y = \overline{Y}$, and (iii) (55) and (56) admit a unique solution $(P^{x'}(\overline{Y}, 0), V^{x'}(\overline{Y}, 0))$ for $x' = A, B$ at $Y = \overline{Y}$.

A.6 Discussion: Numerical Solution Method

The numerical solution requires to solve the system of ODEs from Section A.5. Because the currency values in states $z = A$ and $z = B$ depend on the currency values in state $z = AB$, one has to solve the model backward in terms of the state variable z, starting with state $z = AB$. Having obtained $P^x(Y, AB)$ for $Y \in [0, \overline{Y}]$, one can solve for currency values $P^x(Y, A)$ and $P^x(Y, B)$. Having obtained $P^x(Y, A)$ and $P^x(Y, B)$, one can solve for currency values $P^x(Y, 0)$. In other words, the solution admits the hierarchy: (i) $z = AB$, (ii) $z = A, B$, and (iii) $z = 0$. We start solving the system in the order (i), (ii), and (iii). The solution can be numerically obtained via a standard ODE solver, such ode15s in Matlab.

B Proofs for Section 3

B.1 Proof of Proposition 5

The proof of Proposition 1 proceeds in two steps. First, we derive expressions for the equilibrium currency values P^x. Specifically, we show that

$$P^A = \sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}, \qquad \text{and} \quad P^B = \frac{2\sqrt{Z^B(\hat{Y} + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}},$$
$$P^C = 1 - \sqrt{\frac{Z^A}{\hat{Y} + \pi^A}} - \frac{2\sqrt{Z^B(Y + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}}. \tag{57}$$

Second, we conduct comparative statics in $\hat{Y}$ which then is treated (with some abuse of notation) as parameter.

B.1.1 Equilibrium Quantities

To start with, notice that by market clearing with $\theta = 0$, we have $m^x = P^x$. Next, we take $\hat{Y} := Yv'(m^C)$, $\kappa^x = r_t^x = 0$, $v(m^x) = \frac{(m^x)^{1-\eta}-1}{1-\eta}$ with $\eta = 2$, and we notice that the equilibrium pricing condition (11) simplifies to

$$Z^A(P^A)^{-2} - \pi^A = Z^B(P^B)^{-2} - \frac{\pi^B P^A}{P^B} = \hat{Y}. \tag{58}$$

First, we can solve $Z^A(P^A)^{-2} - \pi^A = \hat{Y}$ to get:

$$P^A = \sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}.$$

Inserting this expression for P^A into (58), we obtain:

$$Z^B(P^B)^{-2} - \left(\frac{\pi^B}{P^B}\right)\sqrt{\frac{Z^A}{\hat{Y} + \pi^A}} = \hat{Y} \quad \Longleftrightarrow \quad Z^B - \pi^B P^B \left(\sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}\right) - \hat{Y}(P^B)^2 = 0.$$

Thus, we have to solve a quadratic equation in P^B, which admits two solutions

$$P^B = \frac{1}{2\hat{Y}}\left[-\pi^B\left(\sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}\right) \pm \sqrt{\frac{Z^A(\pi^B)^2}{\hat{Y} + \pi^A} + 4\hat{Y}Z^B}\right].$$

One solution is clearly negative and thus constitutes no equilibrium. The positive solution can be rewritten as

$$P^B = \frac{1}{2\hat{Y}}\left(\sqrt{\frac{Z^A(\pi^B)^2}{\hat{Y} + \pi^A} + 4\hat{Y}Z^B} - \pi^B\left(\sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}\right)\right). \tag{59}$$

Expression (59) readily implies that P^B increases with π^A, but decreases with Z^A.

Multiplying and dividing both sides of (59) by $\sqrt{\frac{Z^A(\pi^B)^2}{\hat{Y}+\pi^A} + 4\hat{Y}Z^B} + \pi^B\left(\sqrt{\frac{Z^A}{\hat{Y}+\pi^A}}\right)$ and simplifying, one can rewrite (59) as

$$P^B = \frac{2\,Z^B}{\sqrt{\frac{4\,Z^B\,\hat{Y}^2 + 4\,Z^B\,\pi^A\,\hat{Y} + Z^A\,\pi^{B\,2}}{\hat{Y}+\pi^A}} + \pi^B\sqrt{\frac{Z^A}{\hat{Y}+\pi^A}}},$$

which, in turn, can be written as

$$P^B = \frac{2\sqrt{Z^B(\hat{Y} + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\sqrt{Z^A/Z^B}}. \tag{60}$$

Finally, we can use $P^A + P^B + P^C = 1$ to calculate

$$P^C = 1 - \sqrt{\frac{Z^A}{\hat{Y} + \pi^A}} - \frac{2\sqrt{Z^B(\hat{Y} + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\sqrt{Z^A/Z^B}}. \tag{61}$$

The equilibrium is well-defined as long as $P^C \geq 0$, that is, when

$$\sqrt{\frac{Z^A}{\hat{Y}+\pi^A}} + \frac{2\sqrt{Z^B(\hat{Y}+\pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}} \leq 1.$$

holds. Given the explicit closed-form solution, we conclude that, provided its existence, the equilibrium is unique.

B.1.2 Comparative Statics

The corollary follows by direct calculation. We impose $Z^A = Z^B = Z$ to ease the calculations and to simplify the expressions. The expression for P^B in (57) becomes:

$$P^B = \frac{2\sqrt{Z(\hat{Y}+\pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B}.$$

We can then write:

$$\frac{dP^B}{d\hat{Y}} = \frac{\left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B\right)\sqrt{\frac{Z}{\hat{Y}+\pi^A}} - \left(\sqrt{Z(\hat{Y}+\pi^A)}\right)\frac{8\hat{Y}+4\pi^A}{\sqrt{4\,\hat{Y}^2+4\,\pi^A\,\hat{Y}+(\pi^B)^2}}}{\left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B\right)^2}.$$

Note that the denominator of above expression is unambiguously positive. Thus, the sign of the derivative is obtained by inspecting the numerator. The numerator has the same sign as:

$$\left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B\right)\sqrt{\frac{1}{\hat{Y}+\pi^A}} - \sqrt{\hat{Y}+\pi^A}\left(\frac{8\hat{Y}+4\pi^A}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2}}\right),$$

which has the same sign as

$$\left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B\right) - \frac{(8\hat{Y}+4\pi^A)(\hat{Y}+\pi^A)}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2}}.$$

For $\hat{Y} = 0$, the above expression simplifies to:

$$2\pi^B - \frac{4(\pi^A)^2}{\pi^B},$$

which is strictly positive if and only if

$$\pi^B > \sqrt{2}\pi^A.$$

Provided $\pi^B > \sqrt{2}\pi^A$, by continuity in $\hat{Y}$, there exists an interval $[0, \underline{Y})$ with $\underline{Y} > 0$, such that P^B increases with $\hat{Y}$ on $[0, \underline{Y})$.

Finally, observe that

$$\lim_{\hat{Y}\to\infty} \left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B \right) - \frac{(8\hat{Y} + 4\pi^A)(\hat{Y} + \pi^A)}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2}} < 0.$$

Thus, by continuity, $\frac{dP^B}{d\hat{Y}} < 0$ and P^B decreases with $\hat{Y}$ for $\hat{Y}$ sufficiently large.

B.2 Proof of Proposition 3

To begin with recall the equilibrium value expressions from (57). When $\hat{Y} = 0$ and $P^C > 0$ (e.g., $\sqrt{\frac{Z^A}{\pi^A}} + \frac{Z^B}{\pi^B\sqrt{Z^A/\pi^A}} < 1$), then

$$P^A = \sqrt{\frac{Z^A}{\pi^A}} \quad \text{and} \quad P^B = \frac{Z^B}{\pi^B P^A} \tag{62}$$

as well as $P^C = 1 - P^A - P^B$.

We can calculate

$$\frac{\partial P^A}{\partial Z^A} = \frac{1}{2\sqrt{Z^A\pi^A}} \quad \text{and} \quad \frac{\partial P^B}{\partial Z^B} = \frac{1}{\pi^B P^A}. \tag{63}$$

Clearly, $\frac{\partial P^A}{\partial Z^A}$ and $\frac{\partial P^B}{\partial Z^B}$ are decreasing in π^A and π^B respectively.

Next, we observe that:

$$\frac{\partial P^A}{\partial Z^A} - \frac{\partial P^B}{\partial Z^B} = \frac{1}{2\sqrt{Z^A\pi^A}} - \frac{1}{\pi^B P^A} = \frac{1}{2\sqrt{Z^A\pi^A}} - \frac{1}{\pi^B}\sqrt{\frac{\pi^A}{Z^A}},$$

where the second equality uses $P^A = \sqrt{\frac{Z^A}{\pi^A}}$. Multiplying both sides by $2\sqrt{Z^A\pi^A}\pi^B > 0$, we note that $\frac{\partial P^A}{\partial Z^A} - \frac{\partial P^B}{\partial Z^B}$ has the same sign as

$$\pi^B - 2\pi^A.$$

By continuity, when $\pi^B \in (\pi^A, 2\pi^A)$ and $\hat{Y} \geq 0$ is sufficiently low, then $\frac{\partial P^A}{\partial Z^A} < \frac{\partial P^B}{\partial Z^B}$, which we aimed to show.

Next, notice that by (62), P^A is independent of Z^B. As such, we can differentiate the market clearing condition, $P^A + P^B + P^C = 1$ with respect to Z^B to obtain $\frac{\partial P^C}{\partial Z^B} = -\frac{\partial P^B}{\partial Z^B}$. Next, observe that — as discussed before — when $\pi^B \in (\pi^A, 2\pi^A)$ and $\hat{Y} \geq 0$ is sufficiently low, then $\frac{\partial P^B}{\partial Z^B} > \frac{\partial P^A}{\partial Z^A}$, so $-\frac{\partial P^B}{\partial Z^B} < -\frac{\partial P^A}{\partial Z^A}$. Last, differentiation of the market clearing condition, $P^A + P^B + P^C = 1$, with respect to Z^A yields $\frac{\partial P^A}{\partial Z^A} + \frac{\partial P^B}{\partial Z^A} + \frac{\partial P^C}{\partial Z^A} = 0$. Thus,

$$-\frac{\partial P^A}{\partial Z^A} = \frac{\partial P^B}{\partial Z^A} + \frac{\partial P^C}{\partial Z^A} \leq \frac{\partial P^C}{\partial Z^A}.$$

As such, when $\pi^B \in (\pi^A, 2\pi^A)$ and $\hat{Y}$ is sufficiently low, then

$$\frac{\partial P^C}{\partial Z^B} = -\frac{\partial P^B}{\partial Z^B} < -\frac{\partial P^A}{\partial Z^A} \leq \frac{\partial P^C}{\partial Z^A}.$$

This concludes the proof.

Internet Appendix

C Static Model

We present a stylized two-period model which yields similar results to the simplified benchmark from Section 3. We provide an extensive model description so that the interested reader may understand the static model from the Internet Appendix without having to read the model description in the main text.

C.1 Fiat Money in the Two-period Economy

One representative household populates the economy and one generic consumption good serves as the numeraire in which prices are quoted. There are two time periods, $t = 0, 1$, without time discounting. Money serves a combination of the standard roles as: (i) a store of value, (ii) a medium of exchange, and (iii) a unit of account. Two countries, A and B, have their own native fiat currencies A and B. Country $x \in \{A, B\}$ has one unit of currency outstanding whose time-t price is P_t^x in terms of the numeraire.[36]

At $t = 0$, the representative household is endowed with one unit of perishable consumption good. The household only derives consumption utility at $t = 1$, and thus would like to store the endowment from $t = 0$ to $t = 1$. Because the consumption good cannot be stored directly, money serves as a store of value and, specifically, enables the household to use its entire endowment to buy money at $t = 0$ and then sells money at $t = 1$ in exchange for consumption goods. We assume that country x buys back its own currency at $t = 1$ using consumption goods at price P_1^x.[37]

The household can use either currency A or B as a store of value and takes prices as given. Let $m^A \geq 0$ and $m^B \geq 0$ denote the amount of consumption good the household stores at time $t = 0$ in currencies A and B respectively.[38] At $t = 0$, the household invests the whole unit of consumption good in money, i.e., $m^A + m^B = 1$. Denote the time-0 price of currency A by $P^A = P_0^A$. With unit supply, the initial market capitalization of currency x in terms of the numeraire is also P^x. Because the household is the only holder of money, market clearing requires $P^x = m^x$. As a result,

$$P^A + P^B = 1. \tag{64}$$

At $t = 1$, the household sells currency x at price P_1^x and consumes the proceeds, so the household's consumption at $t = 1$ reads: $c = P_1^A + P_1^B$. We call without loss of generality country A "strong" and country B "weak," in that $P_0^A \geq P_0^B$ and currency A serves as the reserve currency at $t = 0$ in a way we make precise shortly.

[36] One could allow an exogenous growth of currency supplies, which does not add further insights to our model.

[37] This is consistent with how a government typically guarantees the value of the currency through its ability to raise real resources via taxation and offer to purchase currency using those resources (Obstfeld and Rogoff, 2017). The dynamic model gets rid of this assumption.

[38] As we demonstrate in the microfoundation of the representative household formulation, $m^A > 0$ and $m^B > 0$ do not imply that all individual agents hold all currencies at the same time.

Household's utility. Money also serves as a medium of exchange (i.e., transaction medium), which we account for in reduced form by stipulating that the household derives a convenience yield from holding money. As such, the household's lifetime utility reads

$$U = c + Z_o[m^A + m^B] + Z^A v(m^A) + Z^B v(m^B), \tag{65}$$

where c is the household's consumption at $t = 1$ and $Z_o(m^A + m^B) + Z^A v(m^A) + Z^B v(m^B)$ is the convenience yield of holding currency from $t = 0$ to $t = 1$.

Crucially, currencies A and B offer different convenience yields $Z_o m^A + Z^A v(m^A)$ and $Z_o m^B + Z^B v(m^B)$, with the difference in convenience captured by the coefficients $Z^A \geq 0$ and $Z^B \geq 0$. For illustration, we take the commonly used CRRA specification with $\eta = 2$:

$$v(m^x) = \frac{(m^x)^{1-\eta} - 1}{1 - \eta} = \frac{m^x - 1}{m^x}. \tag{66}$$

The household derives a constant (marginal) base convenience yield $Z_o > 0$ regardless of whether she holds A or B. The constant Z_o is chosen large enough to ensure that the convenience yield $Z_o m^x + Z^x v(m^x)$ to holding currency x is non-negative in equilibrium and is otherwise immaterial. The functional form (66) has several appealing features. First, as m^x approaches zero, the marginal convenience to holding x becomes arbitrarily large, capturing broadly that x cannot be substituted for certain activities and transactions. As a consequence, $m^x > 0$. Second, as m^x becomes large, the convenience yield to holding currency x diminishes.

Global currency, reserve currency status, and inflation. Both countries must cover expenses, such as the cost of servicing of their outstanding debt or their fiscal deficit. We assume that currency A as the reserve currency is the "global" unit of account in debt contracts and trade invoicing, among other "exorbitant privileges."[39] To capture that international trade invoicing and borrowing are often denominated in dollars in practice, we assume that country x's expenses are denominated in currency A. When country x covers expenses of π^x units of currency A by inflating its currency and reducing the currency value at time $t = 1$, i.e., $P_1^x - P_0^x = \pi^x P^A$, any holder of one unit of consumption good in currency x incurs taxes of $\pi^x(P^A/P^x)$ units of the consumption good, where π^x inversely proxies for the strength of a country's economic fundamentals or a country x's fiscal strength.[40]

We can easily interpret this tax as inflation. Country x's fiscal strength (i.e., π^x) affects inflation

[39] Du et al. (2020) show that countries which are able to issue more domestic currency debt are also the ones that issue more debt denominated in foreign currency; Maggiori et al. (2020, 2019) document that U.S. dollar is the primary currency of denomination (over 60%) since the 2008 crisis in cross-border investors portfolio holdings, even when neither the investor nor the issuer are based in the United States; a dollar dominance similarly manifests in invoicing traded goods (e.g., Goldberg and Tille, 2008; Gopinath and Stein, 2021), consistent with the international use of the dollar as a unit of account (e.g., Matsuyama et al., 1993; Doepke and Schneider, 2017); Gourinchas (2019) and Jiang et al. (2020, 2021), among others, further elaborate on the dollar dominance.

[40] We require π^x to be sufficiently small to ensure positive P_1^x, a restriction relaxed in the dynamic model. The inflation tax does not have to be denoted in the reserve currency either. We introduce a fiscal cost in numeraires in the dynamic setting and most of the results remain robust.

and thus the benefits of holding currency x, which in turn determines the strength and value of currency x. The main purpose of introducing the parameter π^x is to capture this empirically relevant link between a country's fiscal strength or economic fundamentals and the strength of its currency (Jiang et al., 2020). One could also model this link between fiscal strength and currency strength by stipulating that the convenience yield of currency x directly depends on the economic fundamentals of country x. Our results are robust as long as a country's fiscal strength improves the benefits of holding its currency.

C.2 Equilibrium for Traditional Currency Competition

On the margin, the household must be indifferent between allocating funds to currency A and to currency B, subject to $m^A + m^B = 1$. Taking prices P^x as given and considering market clearing:

$$\frac{Z^A}{(P^A)^2} - \pi^A = \frac{Z^B}{(P^B)^2} - \frac{\pi^B P^A}{P^B}, \tag{67}$$

which together with (64) pins down the currency values P^A and P^B. Condition (67) states that in equilibrium, the sum of the marginal convenience yield, $\frac{Z^x}{(m^x)^2} = \frac{Z^x}{(P^x)^2}$, and inflation, π^A and $\pi^B P^A/P^B$ respectively, must be equal across currencies.

Proposition 4. *There exists an equilibrium; a sufficient condition for equilibrium uniqueness is $\pi^B \leq 2Z^B$. In equilibrium, (67) holds. When $\pi^B > \pi^A, Z^A \geq Z^B$ (i.e., country B is weak relative to country A) and the equilibrium is unique, the currency value P^A satisfies $P^A > 1/2 > P^B$. Then, currency A carries less inflation than currency B, in that $\pi^A < \pi^B(P^A/P^B)$.*

C.3 The Rise of Cryptocurrencies

We now add a representative cryptocurrency C with a fixed unit supply. The cryptocurrency C is traded in a frictionless secondary market against the consumption good at price P^C. The household can now store its wealth from $t = 0$ to $t = 1$ by buying cryptocurrencies. We assume, for simplicity that the convenience yield from holding C is $v^C(m^C) = Z_o m^C + \hat{Y} m^C$, where m^C denotes the household's holdings in terms of consumption goods and $\hat{Y} \geq 0$ is a constant. Many studies elaborate on the types of cryptocurrencies and their functions and benefits (e.g., Cong and Xiao, 2021), here we are focusing on general payment tokens such as Bitcoin or Tether that directly competes with fiat currencies.

Cryptocurrency market clearing at $t = 0$ requires $P^C = m^C$. For simplicity, we assume that at $t = 1$, cryptocurrency is traded at the same price P^C in a frictionless secondary market, so households can sell cryptocurrency to the market at price for P^C units of consumption good; the dynamic model gets rid of this assumption.

Notice that implicitly, there are no other ways for country x to cover expenses π^x than imposing a tax on currency holdings. Moreover, unlike government-issued money, cryptocurrency systems do not impose explicit tax and are algorithmically committed to moderate inflation. We incorporate this reality by stipulating that cryptocurrency holdings are not directly taxed.

The crypto equilibrium. Currency competition occurs now within the triangular relationship between countries A and B as well as the cyber economy with the cryptocurrency C, leading to both country-to-country and country-to-cryptocurrency competitions. To characterize the effects of cryptocurrencies, we look now for a "crypto equilibrium," with $m^C > 0$ and $P^C > 0$. Also in the presence of cryptocurrencies, the household stores its entire endowment at $t = 0$ in money, so that $m^A + m^B + m^C = 1$. Market clearing for currency x implies $m^x = P^x$, so that:

$$P^A + P^B + P^C = 1. \tag{68}$$

In the crypto equilibrium, the household is indifferent between exchanging a marginal unit of cryptocurrency for one unit of currency A and B. As we show in Appendix E (which provides the detailed solution to the static model with cryptocurrency), currency values in a crypto equilibrium satisfy:

$$\begin{aligned} P^A &= \sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}, \qquad \text{and} \quad P^B = \frac{2\sqrt{Z^B(\hat{Y} + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}}, \\ P^C &= 1 - \sqrt{\frac{Z^A}{\hat{Y} + \pi^A}} - \frac{2\sqrt{Z^B(\hat{Y} + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}}. \end{aligned} \tag{69}$$

Note that for the crypto equilibrium to exist, it must be that P^C in (69) is positive.

Interestingly, (69) illustrates that the cryptocurrency market acts as a type of buffer zone in the competition between currency A and B. For instance, a decrease in π^B which leads currency B to appreciate causes the cryptocurrency price P^C to fall, but does not affect the price of currency A. In contrast, a decrease in π^A and an appreciation of currency A cause both currency B and cryptocurrencies to depreciate. The underlying reason is that country B's expenses are denominated in terms of currency A. However, the consequences of the appreciation of currency A are partially absorbed by cryptocurrencies. We summarize these findings in the following Proposition.

Proposition 5. *The crypto equilibrium, if it exists (e.g., when $\hat{Y}$ is sufficiently large), is unique. It features $m^x = P^x$, where currency values P^x for $x \in \{A, B, C\}$ are characterized in (69). The value of currency A increases with Z^A, decreases with π^B, and does not depend on Z^B and π^B. The value of currency B decreases with Z^A and π^B, but increases with Z^B and π^A.*

As in Section 3, we obtain Insight 1/

Insight 1: Cryptocurrencies harm strong currency A but may benefit the weaker currency B. The rise of cryptocurrencies unambiguously harms the strong country A and thus the reserve currency A, in that P^A decreases with $\hat{Y}$. The right panel in Figure 13 graphically illustrates this effect by showing that the value of currency A decreases with $\hat{Y}$. Not surprisingly, the

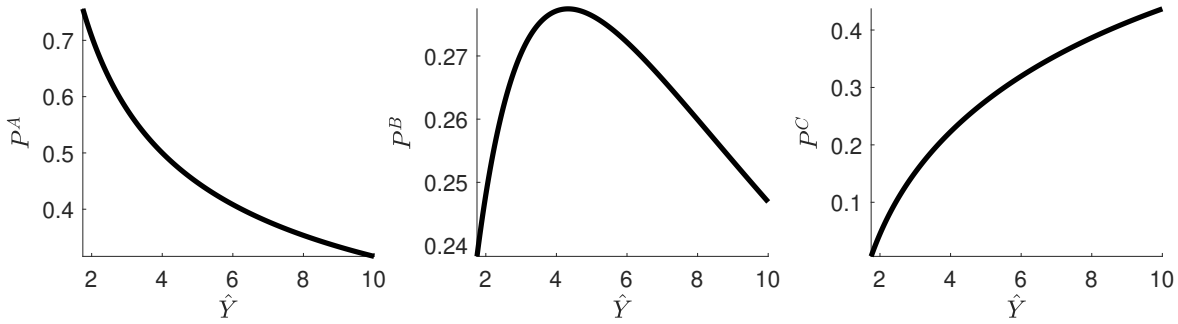

Figure 13: Comparative Statics in a crypto equilibrium with respect to $\hat{Y}$. We set $Z^A = Z^B = 1$, $\pi^A = 0$, and $\pi^B = 5$, which ensures the existence of a (unique) cryptocurrency equilibrium with $P^C > 0$ for all parameters considered.

cryptocurrency value P^C increases with $\hat{Y}$, implying that $\hat{Y}$ quantifies cryptocurrency adoption and the size and value of the cryptocurrency market/sector (or the cyber economy).

The rise of cryptocurrencies may benefit the relatively weaker country and currency, in that P^B follows an inverted U-shaped pattern in $\hat{Y}$ as seen in the middle Panel in Figure 13. Intuitively, the rise of cryptocurrencies mitigates the adverse effects of "dollarization" country B is exposed to, weakening the feedback between currency usage and inflation/depreciation. The cryptocurrency growth (i.e., an increase in $\hat{Y}$) reduces the demand for both currency A and B, thereby decreasing P^A and P^B. However, as currency A depreciates, country B's expenses denominated in currency A fall too, which reduces inflation and benefits currency B. The rise of cryptocurrency weakens currency B as a direct competition but at the same time reduces the degree of competition currency B faces from currency A. When the strong currency is dominant and π^B is sufficiently large compared with π^A, this second effect dominates at low values of $\hat{Y}$, as the following corollary formalizes.

Corollary 1. *Suppose a crypto equilibrium exists. The rise of cryptocurrencies harms the strong currency A, i.e., P^A decreases with $\hat{Y}$. But, the rise may benefit the weak currency B: If and only if $\pi^B > \sqrt{2}\pi^A$, there exists an interval $[0, \underline{Y})$ with $\underline{Y} > 0$ on which P^B increases with $\hat{Y}$. For sufficiently large $\hat{Y}$, P^B decreases with $\hat{Y}$.*

In our framework, banning or regulating cryptocurrencies by any country (or both) can be interpreted as reducing usability and thus the convenience yield $\hat{Y}$ to holding cryptocurrencies. As the currency value of the strong country P^A decreases with $\hat{Y}$, countries with a strong currency benefit the most from banning and regulating the cryptocurrency market.

In contrast, because the currency value of the weak country may increase with $\hat{Y}$, countries with a weak currency benefit less from such regulation or are reluctant to ban and regulate cryptocurrencies at all. Even more, such countries may even want to stimulate cryptocurrency usage within their country, which could be interpreted as an increase of usability and convenience yield $\hat{Y}$. Note that according to Corollary 1, the weak country's currency value increases in $\hat{Y}$ for sufficiently small

values of $\hat{Y} \geq 0$ if and only if the inflation of currency B is sufficiently high ($\pi^B > \sqrt{2}\pi^A$). Countries with very weak currencies (e.g., developing countries) therefore benefit from cryptocurrencies and, possibly, from adopting them as means of payment within their country.

C.4 Central Bank Digital Currencies (CBDCs)

As in the main text, we interpret CBDC in a technology-neutral manner that does not rely on any specific designs. In particular, we view CBDC issuance simply as an increase in the convenience of currency x, i.e., CBDC issuance by country x increases Z^x. As such, $\frac{\partial P^x}{\partial Z^x}$ quantifies (in reduced form) how much currency x benefits from CBDC issuance or country x's incentives to launch CBDC.

The effects and incentives behind CBDC issuance. Implementing CBDC constitutes a way to compete in technology with other (digital) currencies. Depending on the parameter values, in particular that of $\hat{Y}$, the implementation can have a differential impact on fiat-to-fiat and fiat-to-cryptocurrency competitions. To start, note that (69) reveals that CBDC issuance by either country weakens the cryptocurrency value and adoption P^C, in that $\frac{\partial P^C}{\partial Z^x} < 0$. Importantly, sufficiently large values of Z^A and Z^B due to CBDC issuance spell the demise of the crypto sector.

To gain more intuition on the benefits and incentives behind countries' CBDC development, consider a simple case where $\hat{Y} = 0$ and $P^C > 0$ (e.g., $\sqrt{\frac{Z^A}{\pi^A}} + \frac{Z^B}{\pi^B\sqrt{Z^A/\pi^A}} < 1$). Consider $\hat{Y} = 0$. Inserting $\hat{Y} = 0$ into the price expression P^A in (69), we readily obtain $P^A = \sqrt{\frac{Z^A}{\pi^A}}$. Solving (87) with $\hat{Y} = 0$ for P^B is equivalent to solving

$$Z^B(P^B)^{-2} - \left(\frac{\pi^B}{P^B}\right)P^A = 0 \quad \Longleftrightarrow \quad Z^B - \pi^B P^B P^A = 0.$$

for P^B. Thus,

$$P^B = \frac{Z^B}{\pi^B P^A} = \frac{Z^B}{\pi^B\sqrt{Z^A/\pi^A}},$$

where we have used $P^A = \sqrt{\frac{Z^A}{\pi^A}}$.

Next, taking the derivatives with respect to Z^A and Z^B, we get:

$$\frac{\partial P^A}{\partial Z^A} = \frac{1}{2\sqrt{Z^A\pi^A}} \quad \text{and} \quad \frac{\partial P^B}{\partial Z^B} = \frac{1}{\pi^B P^A},$$

which is (63). Now, observe that:

$$\frac{\partial P^A}{\partial Z^A} - \frac{\partial P^B}{\partial Z^B} = \frac{1}{2\sqrt{Z^A\pi^A}} - \frac{1}{\pi^B P^A} = \frac{1}{2\sqrt{Z^A\pi^A}} - \frac{1}{\pi^B}\sqrt{\frac{\pi^A}{Z^A}},$$

where the second equality uses $P^A = \sqrt{\frac{Z^A}{\pi^A}}$. Multiplying both sides by $2\sqrt{Z^A\pi^A}\pi^B > 0$, we note

that $\frac{\partial P^A}{\partial Z^A} - \frac{\partial P^B}{\partial Z^B}$ has the same sign as: $\pi^B - 2\pi^A$.

That is,

$$sign\left(\frac{\partial P^A}{\partial Z^A} - \frac{\partial P^B}{\partial Z^B}\right) = sign(\pi^B - 2\pi^A).$$

Thus, when $\pi^B \in (\pi^A, 2\pi^A)$ and $\hat{Y}$ is sufficiently low, country B benefits more from issuing CBDC than the strong country does, in that $\frac{\partial P^A}{\partial Z^A} < \frac{\partial P^B}{\partial Z^B}$. As a result, CBDC issuance offers the largest advantages for countries with non-dominant but relatively strong currencies, such as China or strong emerging economies like India. These countries should also have the strongest incentives to launch CBDC, which is consistent with the first large scale CBDC launch by China and not the United States.[41]

Again, as in Section 3, we obtain Insight 2.

Insight 2: Country B's CBDC poses a greater threat to cryptocurrencies. Given $\frac{\partial P^C}{\partial Z^B} = -\frac{\partial P^B}{\partial Z^B} < -\frac{\partial P^A}{\partial Z^A} \leq \frac{\partial P^C}{\partial Z^A}$ for $\pi^B < 2\pi^A$, our findings also suggest that CBDC issuance by countries with strong but non-dominant currencies like China or India pose a bigger threat to cryptocurrencies than CBDC issuance by the United States does.[42] The intuition is that cryptocurrencies mainly compete with weaker currencies rather than the reserve currency, so that any appreciation by weaker currencies harms the cryptocurrency market value more.

And, as in Section 3, we also obtain Insight 3.

Insight 3: Pecking order of CBDC issuance. Overall, we observe a pecking order of CBDC issuance. Non-dominant but vibrant emerging economies such as China or India, benefit the most from implementing CBDC, followed by the strong countries such as the United States that are already dominant in the global currency competition. Countries with very weak currencies (e.g., $\pi^B > 2\pi^A$), such as El Salvador, benefit the least from CBDC issuance, because $\frac{\partial P^B}{\partial Z^B}$ decreases with π^B. Intuitively, the currency of these countries is weak regardless of the implementation of CBDC, and CBDC issuance by such countries has negligible impact on the strong country's currency or the cryptocurrency market. As mentioned earlier, these countries may find it advantageous to directly adopt non-pegged cryptocurrencies as legal means of payment within their territory.

[41] The key motivations of China for introducing eCN are cited as limiting the dominance of private payment services. However, both mobile service provision and eCN, once more international, can challenge U.S. dollars and Euros. After all, eCN technology likely opens commercial opportunities for China in some emerging markets, amplifying China's influence in emerging economies, something U.S. and EU foreign policy experts may have to consider.

[42] For a derivation, notice that by (69), P^A is independent of Z^B in a crypto equilibrium. As such, we can differentiate the market clearing condition, $P^A + P^B + P^C = 1$ with respect to Z^B to obtain $\frac{\partial P^C}{\partial Z^B} = -\frac{\partial P^B}{\partial Z^B}$. Next, observe that — as discussed before — when $\pi^B \in (\pi^A, 2\pi^A)$, then $\frac{\partial P^B}{\partial Z^B} > \frac{\partial P^A}{\partial Z^A}$, so $-\frac{\partial P^B}{\partial Z^B} < -\frac{\partial P^A}{\partial Z^A}$. Last, differentiation of the market clearing condition, $P^A + P^B + P^C = 1$, with respect to Z^A yields $\frac{\partial P^A}{\partial Z^A} + \frac{\partial P^B}{\partial Z^A} + \frac{\partial P^C}{\partial Z^A} = 0$. Thus,

$$-\frac{\partial P^A}{\partial Z^A} = \frac{\partial P^B}{\partial Z^A} + \frac{\partial P^C}{\partial Z^A} \leq \frac{\partial P^C}{\partial Z^A},$$

where the inequality uses $\frac{\partial P^B}{\partial Z^A} \leq 0$.

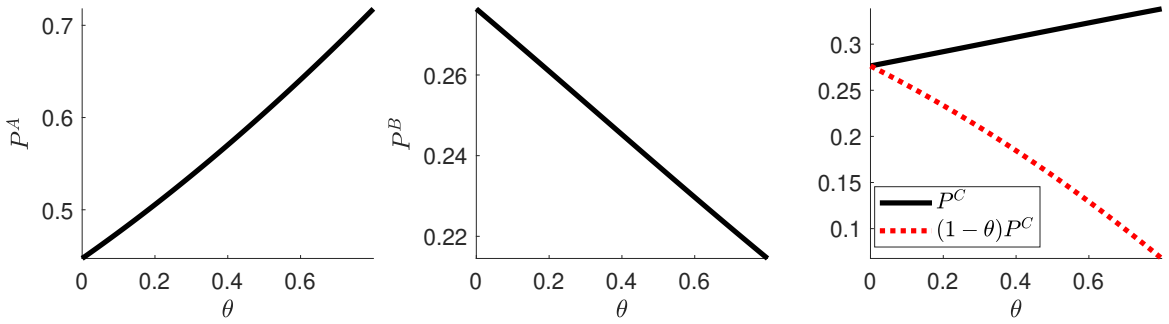

Figure 14: Comparative Statics in a crypto equilibrium with respect to collateralization ratio, θ. The parameterization follows Figure 13. In addition, we set $\hat{Y} = 5$. Under these parameters, a (unique) crypto equilibrium exists.

C.5 Stablecoins and Fiat-backed Cryptocurrency

Our static model can accommodate that some cryptocurrencies, especially stablecoins, are partially backed by the dominant national currency A (i.e., U.S. dollars). Suppose a fraction $\theta \in [0, 1)$ of aggregate cryptocurrency value P^C is backed by currency A, i.e., empirically, θ can be seen as the fraction of aggregate cryptocurrency market capitalization that stems from U.S. dollar backed stablecoins. In that case, $\theta P^C/P^A$ units of currency A are kept as reserves backing cryptocurrency and thus are locked up, which leaves $1-\theta P^C/P^A$ units of currency A as the circulating supply held by the household. That is, $m^A = P^A(1 - \theta P^C/P^A) = P^A - \theta P^C$, while $m^B = P^B$ and $m^C = P^C$, which implies the market clearing condition:

$$P^A(1 - \theta P^C/P^A) + P^B + P^C = 1 \quad \iff \quad P^A + P^B + P^C(1 - \theta) = 1. \tag{70}$$

For simplicity, we do not consider that the degree of reserves backing cryptocurrency affects the convenience yield to holding cryptocurrency.[43] Internet Appendix G presents the solution to this model extension with fiat-backed cryptocurrencies, and solves for currency values P^A, P^B, and P^C in closed-form. Figure 14 plots the equilibrium currency values P^A (left panel), P^B (middle panel), and P^C and $P^C(1 - \theta)$ (right panel) against θ. Both the value of currency A and cryptocurrency C increase with θ, while P^B decreases with θ. In addition, the market value of cryptocurrency in excess of its reserves, $(1 - \theta)P^C$, decreases with θ.

Intuitively, if cryptocurrencies are (partially) backed by reserves consisting of currency A (or assets denominated in currency A), demand for cryptocurrencies also stimulates demand for currency A. Put differently, the seigniorage from cryptocurrency usage partially accrues to country A which in turn harnesses part of the cryptocurrency convenience yield. This effect implies that a higher collateralization ratio θ raises demand for currency A and therefore currency value P^A, i.e.,

[43] Admittedly, in practice, reserves backing cryptocurrency could have ambiguous effects. For instance, a higher level of reserves backing a stablecoin improves its stability, which is beneficial to users, but may come at the expense of higher fees and a reduced degree of decentralization. Moreover, the level of reserves also affects the profitability of stablecoin issuers, which endogenously affects their incentives to develop and to issue stablecoins in the first place.

P^A increases with θ (left panel). At the same time, a stronger currency A exacerbates competition for currency B, so that the value of currency B falls with θ (middle panel).

Interestingly, the cryptocurrency market value also benefits from being backed by reserves of currency A, in that P^C increases with θ. The underlying reason is that an increase in θ strengthens currency A and, because some of country B's expense are denominated in currency A, raises the inflation of currency B. The increase in inflation, in turn, makes households substitute their holdings of currency B toward currency A and cryptocurrency. However, the actual seigniorage revenue accruing to the issuer of cryptocurrency is only $(1-\theta)P^C$ units of the consumption goods, because θP^C units of the consumption are used to build reserves (i.e., as collateral). As Panel C illustrates, the seigniorage captured by the cryptocurrency sector decreases with θ, as A now seizes part of the seigniorage generated by cryptocurrencies.

Analogously to the findings in Section C.5, we obtain Insight 4.

Insight 4: Regulated stablecoins as digital dollar. These findings generate insights regarding the benefits, risk, and regulation of (U.S. dollar) stablecoins. Prominently, requiring stablecoins pegged to the U.S. dollar to be backed by U.S. dollar assets can strengthen the dominance of the U.S. dollar, while weakening other national currencies. When stablecoins are backed by U.S. dollar assets, part of the seigniorage created by the cryptocurrency accrue to the United States. U.S. dollar stablecoins can effectively export a digital version of the U.S. dollar to other countries or the digital economy in which cryptocurrency is adopted, possibly increasing the "reach" and global influence (and exorbitant privilege) of the U.S. dollar.

As a result, regulation that restricts or bans stablecoin issuance may not be optimal for the United States. Instead, the U.S and government could benefit from regulation that requires stablecoin issuers to hold U.S. dollar reserves, so as to reclaim seigniorage from the cryptocurrency sector and to benefit from the adoption of these stablecoins. Facilitating regulated issuance of U.S. dollar stablecoins, the U.S. could "delegate" the creation of a digital dollar to the private sector, whilst capturing part of the generated seigniorage revenues.[44]

D Omitted Proofs and Technical Details for the Static Model

D.1 Proof of Proposition 4

Part I discusses the household optimization, and derives the equilibrium condition (67). Part II establishes existence of the equilibrium, and provides a sufficient condition for its uniqueness.

We define $v^x(m^x) \equiv Z_o m^x + Z^x v(m^x)$ for $x = A, B$, with the function $v(m^x)$ defined in (66).

[44] More broadly, requiring cryptocurrencies and digital payment systems to use a fiat currency or CBDC as collateral or reserve would have a similar effect as the stablecoin here. Given that digital payment systems such as Alipay enjoy a liquidity premium as money or treasury debt do (Chen and Jiang, 2022), our analysis provides insights on how they affect currency competition.

D.1.1 Part I — Household Optimization

At time $t = 0$, the household acquires m^x/P_0^x units of currency x which equals m^x units of currency x in terms of the consumption good. At time $t = 1$, the household sells m^x/P_0^x units of currency x at price P_1^x and consumes the proceeds. Thus, total consumption at time $t = 1$ reads

$$c = \frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} \tag{71}$$

As the household does not derive any utility from consuming at time $t = 0$, it invests its entire endowment in money at time $t = 0$, so $m^A + m^B = 1$.

Recall the household optimizes lifetime utility in (65), i.e., the representative household solves:

$$\max_{m^A, m^B \geq 0} \left(\frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} + v^A(m^A) + v^B(m^B) \right) \quad \text{s.t.} \quad m^A + m^B = 1, \tag{72}$$

taking prices P_t^A and P_t^B as given, where we inserted consumption c at time $t = 1$ characterized in (71). We now can insert $m^A + m^B = 1 \iff m^B = 1 - m^A$ into the objective in (72) and rewrite the objective in (72) as:

$$\max_{m^A \in [0,1]} \left(\frac{P_1^A m^A}{P_0^A} + \frac{P_1^B (1 - m^A)}{P_0^B} + v^A(m^A) + v^B(1 - m^A) \right). \tag{73}$$

Provided $m^A \in (0, 1)$ is interior, the following first order condition with respect to m^A must hold:

$$\frac{P_1^A}{P_0^A} + \frac{\partial}{\partial m^A} v^A(m^A) = \frac{P_1^B}{P_0^B} - \frac{\partial}{\partial m^A} v^B(1 - m^A). \tag{74}$$

The second order condition to (73), i.e.,

$$\frac{\partial^2}{\partial (m^A)^2} \left(v^A(m^A) + v^B(1 - m^A) \right) < 0, \tag{75}$$

must hold for an interior maximum m^A. Since $v^x(m^x) = Z_o m^x + v(m^x)$ and $v(m^x)$ is strictly concave, the second order condition (75) becomes:

$$v''(m^A) + v''(1 - m^A) < 0.$$

As the second order condition is satisfied, the first order condition (74) is sufficient. We now consider the interior equilibrium, i.e., $m^x \in (0, 1)$; we verify that, indeed, under the assumed functional forms and parameter conditions, the equilbrium features $m^x = P^x \in (0, 1)$.

Next, notice that

$$P_1^x = P_0^x - \pi^x P_0^A,$$

so that

$$\frac{P_1^A}{P_0^A} = 1 - \pi^A \quad \text{and} \quad \frac{P_1^B}{P_0^B} = 1 - \frac{\pi^B P_0^A}{P_0^B}.$$

Using these relations, we can rewrite the equilibrium first order condition (74) as

$$\frac{\partial}{\partial m^A} v^A(m^A) - \pi^A = -\frac{\partial}{\partial m^A} v^B(1 - m^A) - \frac{\pi^B P_0^A}{P_0^B}. \tag{76}$$

(76) simplifies after substituting $v(m^x)$ in (66) into $\frac{\partial}{\partial m^A} v^A(m^A) = Z_o + \frac{\partial}{\partial m^A} v(m^A)$ and $-\frac{\partial}{\partial m^A} v^B(1 - m^A) = Z_o - \frac{\partial}{\partial m^A} v(1 - m^A)$ and using $m^x = P^x$ and $P^A + P^B = 1$ (so $1 - P^A = P^B$):[45]

$$Z^A (P^A)^{-2} - \pi^A = Z^B (P^B)^{-2} - \frac{\pi^B P^A}{P^B}. \tag{77}$$

Condition (77) is equivalent to

$$(P^B)^2 [Z^A - \pi^A (P^A)^2] = (P^A)^2 [Z^B - \pi^B P^A P^B].$$

Inserting $P^B = 1 - P^A$ into (77), we obtain

$$Z^A (P^A)^{-2} - \pi^A = Z^B (1 - P^A)^{-2} - \frac{\pi^B P^A}{1 - P^A}, \tag{78}$$

which is the equilibrium condition (67) in terms of only P^A. To characterize an interior equilibrium, it therefore suffices to solve (78) for $P^A \in (0, 1)$.

It follows that the left-hand-side of (78) tends to $+\infty$ as P^A goes to zero, while the right-hand-side remains finite. Likewise, the right-hand-side of (78) tends to $+\infty$ as P^A goes to one, while the left-hand-side remains finite. As such, there cannot exist an equilibrium with $P^A = 0$ or $P^A = 1$, i.e., the equilibrium, provided it exists, must be interior featuring $m^x = P^x \in (0, 1)$.

D.1.2 Part II — Existence and Uniqueness

For $P^A \in (0, 1)$, define

$$f(P^A) = Z^A (P^A)^{-2} - \pi^A - Z^B (1 - P^A)^{-2} + \frac{\pi^B P^A}{1 - P^A},$$

which is the difference between the right-hand-side and the left-hand-side of (78). According to (78), $f(P^A) = 0$ in equilibrium. It can be seen that $\lim_{P^A \to 1} f(P^A) = -\infty$ and $\lim_{P^A \to 0} f(P^A) = +\infty$. By continuity, there exists a root P^A with $f(P^A) = 0$, i.e., there exists an equilibrium with price P^A.

[45] Note that $v'(m^x) = (m^x)^{-2}$.

The equilibrium is unique if and only if $f(P^A)$ has a unique root in $(0,1)$. We can express:

$$f'(P^A) = -2Z^A(P^A)^{-3} - 2Z^B(1-P^A)^{-3} + \frac{\pi^B}{1-P^A} + \frac{\pi^B P^A}{(1-P^A)^2}.$$

We can multiply $f'(P^A)$ by $(1-P^A)^2$ to obtain:

$$(1-P^A)^2 f'(P^A) = -2Z^A(P^A)^{-3}(1-P^A)^2 - 2Z^B(1-P^A)^{-1} + \pi^B.$$

For $P^A \in (0,1)$, we obtain:

$$(1-P^A)^2 f'(P^A) < \pi^B - 2Z^B.$$

Thus, if

$$\pi^B \leq 2Z^B, \tag{79}$$

then $(1-P^A)^2 f'(P^A) < 0$ and $f(P^A)$ strictly decreases in P^A on $(0,1)$, implying equilibrium uniqueness. As such, (79) is a sufficient condition for equilibrium uniqueness.

Suppose that the equilibrium is unique. Then, when $\pi^B > \pi^A$ and $Z^A \geq Z^B$, we have for $P^A \leq 1/2$ that

$$f(1/2) = \pi^B - \pi^A + 4(Z^A - Z^B) > 0.$$

Given the uniqueness, equilibrium price satisfies $P_A^0 = P^A > 1/2$, which implies via market clearing $P_0^B = P^B < 1/2$. Consequently, $\pi_A < \pi_B P_0^A / P_0^B$, which concludes the argument.

E Proof of Proposition 5

Part I discusses the household optimization, and derives the (necessary) equilibrium condition (86). Part II discusses existence and uniqueness of the equilibrium, when $\eta = 2$ in (66), and also characterizes currency values in closed-form. For the proof, we define $v^x(m^x) \equiv Z_o m^x + Z^x v(m^x)$ for $x = A, B$, with the function $v(m^x)$ defined in (66). We also set $v^C(m^C) \equiv (Z_o + \hat{Y})m^C$.

E.1 Part I — Household Optimization

We start by discussing the representative household's optimization. First, note that at time $t = 0$, the household acquires m^x/P_0^x units of currency x which equals m^x units of currency x in terms of the consumption good. At time $t = 1$, the household sells m^x/P_0^x units of currency x at price P_1^x and consumes the proceeds. Thus, consumption at time $t = 1$ is:

$$c = \frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} + m^C, \tag{80}$$

where we used that $P_0^C = P_1^C$ (i.e., cryptocurrency is traded without friction or cost at the same price at times $t = 0$ or $t = 1$, so there is no "inflation" for cryptocurrency). The lifetime utility of

the representative household is:

$$c + v^A(m^A) + v^B(m^B) + v^C(m^C) \tag{81}$$

As the household does not derive any utility from consuming at time $t = 0$, it invests its entire endowment in money at time $t = 0$, so $m^A + m^B + m^C = 1$.

The household maximizes lifetime utility in (81), that is, the household solves

$$\max_{m^A,m^B,m^C\geq 0} \left(\frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} + m^C + v^A(m^A) + v^B(m^B) + v^C(m^C) \right) \quad \text{s.t.} \quad m^A + m^B + m^C = 1, \tag{82}$$

taking prices P_t^A, P_t^B, and P_t^C as given. We can substitute $m^C = 1 - m^A - m^B$ and rewrite (82) as

$$\max_{m^A,m^B\geq 0} \left(\frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} + 1 - m^A - m^B + v^A(m^A) + v^B(m^B) + v^C(1 - m^A - m^B) \right), \tag{83}$$

subject to $m^C \geq 0 \iff m^A + m^B \leq 1$.

In optimum when $m^A + m^B \in (0, 1)$ and $m^x \in (0, 1)$, the following two first order conditions (with respect to m^A and m^B) must hold:[46]

$$\frac{P_1^A}{P_0^A} - 1 + \frac{\partial}{\partial m^A}\left[v^A(m^A) + v^C(1 - m^A - m^B)\right] = 0 \tag{84}$$

$$\frac{P_1^B}{P_0^B} - 1 + \frac{\partial}{\partial m^B}\left[v^B(m^B) + v^C(1 - m^A - m^B)\right] = 0. \tag{85}$$

We know that $P_1^A/P_0^A = 1 - \pi^A$ and $P_1^B/P_0^B = 1 - \pi^B P_0^A/P_0^B$. Inserting these relations and $m^x = P^x$ into (84), we obtain

$$1 - \pi^A + \frac{\partial}{\partial m^A}\left[v^A(m^A) + v^C(1 - m^A - m^B)\right] = 0$$

$$1 - \frac{\pi^B P_0^A}{P_0^B} + \frac{\partial}{\partial m^B}\left[v^B(m^B) + v^C(1 - m^A - m^B)\right] = 0.$$

Using the explicit expressions for $v^x(m^x) = Z_o m^x + Z^x v(m^x)$ for $x = A, B$ with $v(m^x)$ from (66) and $v^C(m^C) = m^C(Z_o + \hat{Y})$ and doing some algebra, we then obtain (for $P^x = P_0^x$)

$$Z^A(m^A)^{-2} - \pi^A = Z^B(m^B)^{-2} - \frac{\pi^B P^A}{P^B} = \hat{Y}, \tag{86}$$

which becomes after inserting $m^x = P^x$:

$$Z^A(P^A)^{-2} - \pi^A = Z^B(P^B)^{-2} - \frac{\pi^B P^A}{P^B} = \hat{Y}. \tag{87}$$

[46] As before, in the solution to the household's problem in Appendix D.1.1, one can verify that the first order conditions are sufficient.

In Part II below, we combine (86) and (68) to solve for currency values in closed-form.

E.2 Part II — Existence and Uniqueness

With $\eta = 2$ for $v(m^x)$, suppose there exists a cryptocurrency equilibrium, which is characterized by (86). Then

$$Z^A(P^A)^{-2} - \pi^A = Z^B(P^B)^{-2} - \frac{\pi^B P^A}{P^B} = \hat{Y}$$

holds. First, we can solve $Z^A(P^A)^{-2} - \pi^A = \hat{Y}$ to get:

$$P^A = \sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}.$$

Inserting this expression for P^A into (86), we obtain:

$$Z^B(P^B)^{-2} - \left(\frac{\pi^B}{P^B}\right)\sqrt{\frac{Z^A}{\hat{Y} + \pi^A}} = \hat{Y} \quad \Longleftrightarrow \quad Z^B - \pi^B P^B \left(\sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}\right) - \hat{Y}(P^B)^2 = 0.$$

Thus, we have to solve a quadratic equation in P^B, which admits two solutions

$$P^B = \frac{1}{2\hat{Y}}\left[-\pi^B\left(\sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}\right) \pm \sqrt{\frac{Z^A(\pi^B)^2}{\hat{Y} + \pi^A} + 4\hat{Y}Z^B}\right].$$

One solution is clearly negative and thus constitutes no equilibrium. The positive solution can be rewritten as

$$P^B = \frac{1}{2\hat{Y}}\left(\sqrt{\frac{Z^A(\pi^B)^2}{\hat{Y} + \pi^A} + 4\hat{Y}Z^B} - \pi^B\left(\sqrt{\frac{Z^A}{\hat{Y} + \pi^A}}\right)\right). \tag{88}$$

Expression (88) readily implies that P^B increases with π^A, but decreases with Z^A.

Multiplying and dividing both sides of (88) by $\sqrt{\frac{Z^A(\pi^B)^2}{\hat{Y}+\pi^A} + 4\hat{Y}Z^B} + \pi^B\left(\sqrt{\frac{Z^A}{\hat{Y}+\pi^A}}\right)$ and simplifying, one can rewrite (88) as

$$P^B = \frac{2\,Z^B}{\sqrt{\frac{4\,Z^B\,\hat{Y}^2 + 4\,Z^B\,\pi^A\,\hat{Y} + Z^A\,\pi^{B^2}}{\hat{Y}+\pi^A}} + \pi^B\sqrt{\frac{Z^A}{\hat{Y}+\pi^A}}},$$

which, in turn, can be written as

$$P^B = \frac{2\sqrt{Z^B(\hat{Y} + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\sqrt{Z^A/Z^B}}. \tag{89}$$

Finally, we can use $P^A + P^B + P^C = 1$ to calculate

$$P^C = 1 - \sqrt{\frac{Z^A}{\hat{Y} + \pi^A}} - \frac{2\sqrt{Z^B(\hat{Y} + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}}. \tag{90}$$

The crypto equilibrium exists as long as $P^C \geq 0$, that is, when

$$\sqrt{\frac{Z^A}{\hat{Y} + \pi^A}} + \frac{2\sqrt{Z^B(\hat{Y} + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2(Z^A/Z^B)} + \pi^B\,\sqrt{Z^A/Z^B}} \leq 1$$

holds. Given the explicit closed-form solution, we conclude that, provided its existence, the cryptocurrency equilibrium is unique.

F Proof of Corollary 1

The corollary follows by direct calculation. We impose $Z^A = Z^B = Z$ to ease the calculations and to simplify the expressions. The expression for P^B in (69) becomes:

$$P^B = \frac{2\sqrt{Z(\hat{Y} + \pi^A)}}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B}.$$

We can then write:

$$\frac{dP^B}{d\hat{Y}} = \frac{\left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B\right)\sqrt{\frac{Z}{\hat{Y}+\pi^A}} - \left(\sqrt{Z(\hat{Y} + \pi^A)}\right)\frac{8\hat{Y}+4\pi^A}{\sqrt{4\,\hat{Y}^2+4\,\pi^A\,\hat{Y}+(\pi^B)^2}}}{\left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B\right)^2}.$$

Note that the denominator of above expression is unambiguously positive. Thus, the sign of the derivative is obtained by inspecting the numerator. The numerator has the same sign as:

$$\left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B\right)\sqrt{\frac{1}{\hat{Y} + \pi^A}} - \sqrt{\hat{Y} + \pi^A}\left(\frac{8\hat{Y} + 4\pi^A}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2}}\right),$$

which has the same sign as

$$\left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B\right) - \frac{(8\hat{Y} + 4\pi^A)(\hat{Y} + \pi^A)}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2}}.$$

For $\hat{Y} = 0$, the above expression simplifies to:

$$2\pi^B - \frac{4(\pi^A)^2}{\pi^B},$$

which is strictly positive if and only if

$$\pi^B > \sqrt{2}\pi^A.$$

Provided $\pi^B > \sqrt{2}\pi^A$, by continuity in $\hat{Y}$, there exists an interval $[0, \underline{Y})$ with $\underline{Y} > 0$, such that P^B increases with $\hat{Y}$ on $[0, \underline{Y})$.

Finally, observe that

$$\lim_{\hat{Y}\to\infty} \left(\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2} + \pi^B \right) - \frac{(8\hat{Y} + 4\pi^A)(\hat{Y} + \pi^A)}{\sqrt{4\,\hat{Y}^2 + 4\,\pi^A\,\hat{Y} + (\pi^B)^2}} < 0.$$

Thus, by continuity, $\frac{dP^B}{d\hat{Y}} < 0$ and P^B decreases with $\hat{Y}$ for $\hat{Y}$ sufficiently large.

G Solution with Fiat-Backed Cryptocurrency/Stablecoin

We solve the model extension with fiat-backed cryptocurrency. Suppose that fraction $\theta \in [0, 1]$ of cryptocurrency is backed by currency A, where $\theta < \hat{Y}/\pi^B$. That is, total reserves backing the cryptocurrency are θP^C units of the consumption good. Thus, the reserves backing cryptocurrency consist of $\theta P^C/P^A$ units of currency A, which implies a circulating supply of currency A at $(1 - \theta P^C/P^A)$ units. For the market for currency A to clear, the household holds the remainder, i.e., the circulating supply

$$m^A/P^A = (1 - \theta P^C/P^A) \tag{91}$$

of units of currency A. As a consequence, the household's holdings of currency A in units of the consumption good is:

$$m^A = P^A - \theta P^C. \tag{92}$$

With $m^B = P^B$ and $m^C = P^C$, the market clearing condition $m^A + m^B + m^C = 1$ therefore becomes

$$P^A + P^B + P^C(1 - \theta) = 1.$$

Thus, we can solve for

$$P^C = \frac{1 - P^A - P^B}{1 - \theta}. \tag{93}$$

and, inserting P^C from (93) into (92), we solve for:

$$m^A = P^A - \theta P^C = P^A - \frac{\theta(1 - P^A - P^B)}{1 - \theta} = \frac{P^A - \theta(1 - P^B)}{1 - \theta}. \tag{94}$$

As in the baseline, the household is taxed for holding currency x, in that $P_1^x - P_0^x = -\pi^x P_0$, implying $P_1^A/P_0^A = 1 - \pi^A$ and $P_1^B/P_0^B = 1 - \pi^B P_0^A/P_0^B$.

Similar to (82), the household maximizes

$$\max_{m^A,m^B,m^C\geq 0} \left(\frac{P_1^A m^A}{P_0^A} + \frac{P_1^B m^B}{P_0^B} + m^C + v^A(m^A) + v^B(m^B) + v^C(m^C) \right) \quad \text{s.t.} \quad m^A + m^B + m^C = 1,$$

with $v^x(m^x) = Z_o m^x + v(m^x)$ for $x = A, B$ and $v^C(m^C) = (Z_o + \hat{Y})m^C$. As before, one can show that in a cryptocurrency equilibrium with positive price $P^C > 0$, the indifference conditions (86) must hold:

$$Z^A(m^A)^{-2} - \pi^A = Z^B(m^B)^{-2} - \frac{\pi^B P^A}{P^B} = \hat{Y}. \tag{95}$$

Intuitively, the household must be indifferent between substituting one marginal unit of currency x with a marginal unit of another currency. The derivations are analogous to the ones presented in Appendix E.1.

After inserting $m^A = \frac{P^A - \theta(1-P^B)}{1-\theta}$ from (94) and $m^B = P^B$, we obtain

$$Z^A \left(\frac{P^A - \theta(1-P^B)}{1-\theta} \right)^{-2} - \pi^A = \hat{Y}, \tag{96}$$

and

$$Z^B(P^B)^{-2} - \frac{\pi^B P^A}{P^B} = \hat{Y}. \tag{97}$$

The equilibrium is obtained by solving (96), (97), and (93) for P^A, P^B, and P^C.

To solve this system, note that one can solve for (96) and (97), which do not depend on P^C, for P^A and P^B and then plug the solution into (93) to obtain P^C. To begin with, use (97) to solve for

$$P^A = \frac{Z^B/P^B - \hat{Y}P^B}{\pi^B}, \tag{98}$$

and insert this expression into (96) to obtain after rearranging:

$$\left(\frac{Z^B/P^B - \hat{Y}P^B}{\pi^B} - \theta(1-P^B) \right)^{-2} = \frac{\hat{Y} + \pi^A}{Z^A(1-\theta)^2}.$$

Thus,

$$Z^B - \hat{Y}(P^B)^2 - \theta\pi^B(1-P^B)P^B = P^B(1-\theta)\cdot\pi^B\sqrt{\frac{Z^A}{\hat{Y}+\pi^A}}. \tag{99}$$

Define

$$K := \pi^B\sqrt{\left(\frac{Z^A}{\hat{Y}+\pi^A}\right)},$$

and rewrite (99) as:

$$Z^B - P^B\big(K(1-\theta) + \theta\pi^B\big) + (P^B)^2(\theta\pi^B - \hat{Y}) = 0. \tag{100}$$

Equation (100) admits two solutions, if they exist:

$$\frac{K(1-\theta) + \pi^B\,\theta \pm \sqrt{\big(K(1-\theta) + \theta\pi^B\big)^2 - 4Z^B(\theta\pi^B - \hat{Y})}}{-2\,\left(\hat{Y} - \pi^B\,\theta\right)}.$$

$\hat{Y} > \pi^B\theta$ rules out the negative solution. So we get:

$$P^B = \frac{-K(1-\theta) - \pi^B\,\theta + \sqrt{\big(K(1-\theta) + \theta\pi^B\big)^2 - 4Z^B(\theta\pi^B - \hat{Y})}}{2\,\left(\hat{Y} - \pi^B\,\theta\right)}. \tag{101}$$

Inserting P^B into (98), we can derive P^A in closed-form, and, inserting P^A and P^B into (93), we obtain P^C in closed-form. A crypto equilibrium exists if and only if the resulting solution satisfies $P^C \geq 0$. Given the explicit closed-form solution, we conclude that the crypto equilibrium, when exists, is unique.

Finally, note that at time $t = 0$, the cryptocurrency sector collects P^C units of the consumption good from households. Out of these revenues, θP^C units of the consumption good are used to buy currency A which is the reserve backing cryptocurrency. As such, the actual seigniorage revenue of the cryptocurrency sector is $(1-\theta)P^C$.